INVESTIGATION 3

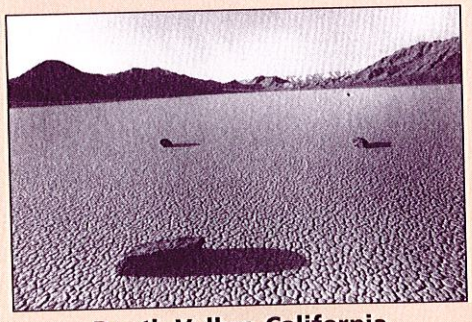
Subtracting Integers

In the last investigation, you used number lines and chip boards to help you learn about the addition of integers. These tools are also helpful for modeling subtraction of integers. In this investigation, you will start by using a chip board to explore subtraction. Next, you will use the relationship between addition and subtraction to subtract numbers on the number line. Finally, you will study patterns involving subtraction of integers and use these patterns to make predictions.

Think about this!

You can use positive and negative numbers to describe elevations. If you think of sea level as 0 feet, you can express elevations above sea level with positive numbers and elevations below sea level with negative numbers.

The highest point in the United States is Mount McKinley (also known as Denali), Alaska, with an elevation of 20,320 feet above sea level. You can express this elevation as $^{+}20{,}320$ feet. The lowest point in the United States is Death Valley, California, with an elevation of 282 feet below sea level. You can express this elevation as $^{-}282$ feet.



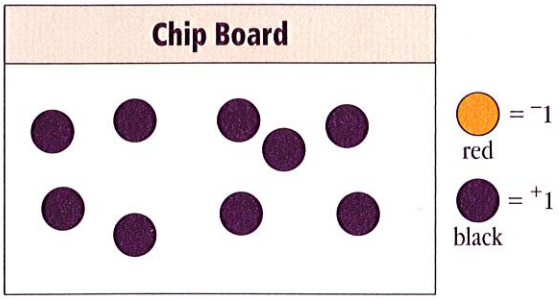
Death Valley, California

How many feet higher is the highest point in the United States than the lowest point?

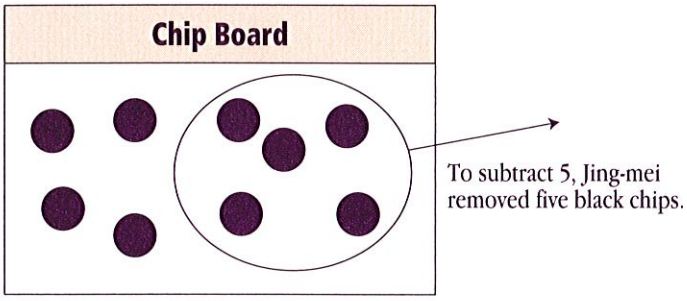
3.1 Subtracting on a Chip Board

Amber's friends Jing-mei and Drew liked Amber's chip board model for adding integers. They decided to use a chip board to explore subtracting integers.

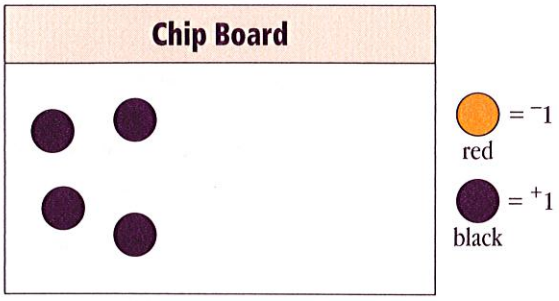
To model 9 – 5, Jing-mei started with an empty chip board and then put nine black chips on the board to represent $^{+}9$.



Jing-mei thinks about subtracting as "taking away." Therefore, to represent subtracting 5, she *removed* five black chips from the board.

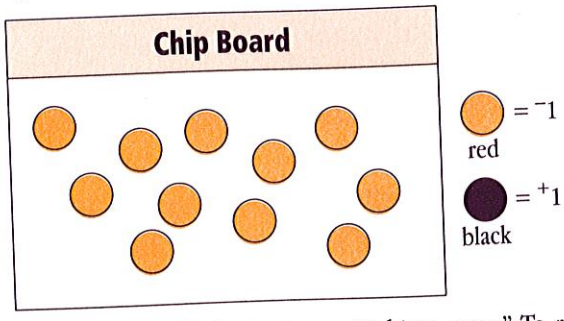


After removing the five black chips, four black chips remained.

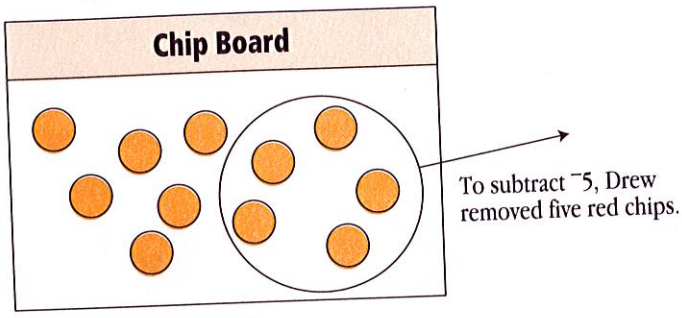


Jing-mei wrote the number sentence 9 – 5 = 4 to represent her work on the chip board.

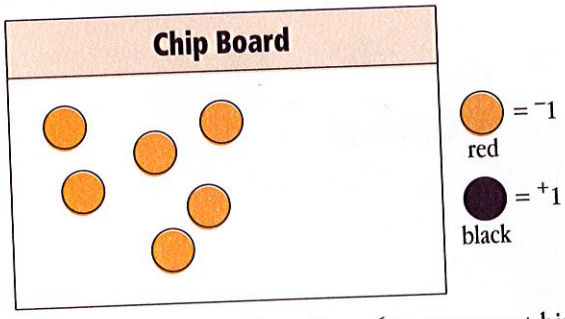
Drew tried Jing-mei's method to find $^{-}11 - {}^{-}5$. He started with an empty board and then put on 11 red chips to represent $^{-}11$.



Like Jing-mei, Drew thought of subtracting as "taking away." To represent subtracting $^{-}5$, he removed five red chips from the board.



Six red chips remained on the board.



Drew wrote the number sentence $^{-}11 - {}^{-}5 = {}^{-}6$ to represent his work on the chip board.

Think about this!

Why does it makes sense that the difference between 9 and 5 is 4 (that is, $9 - 5 = 4$) and the difference between $^{-}11$ and $^{-}5$ is $^{-}6$ (that is, $^{-}11 - {}^{-}5 = {}^{-}6$)?

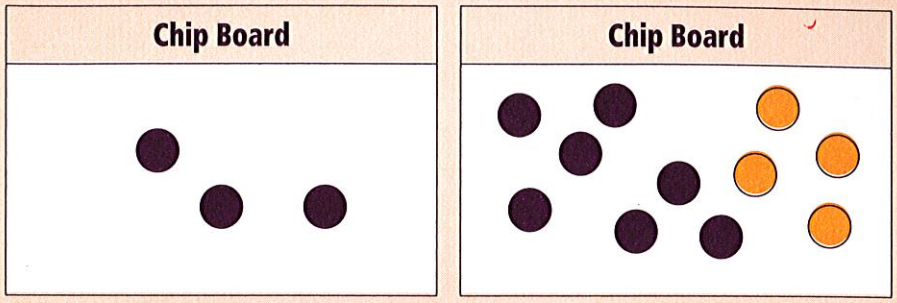
Problem 3.1

A. Use a chip board and black and red chips to find each sum or difference.

1. $^{-}8 - {}^{-}7$ **2.** $^{+}8 + {}^{-}7$

3. $^{-}6 - {}^{-}2$ **4.** $^{+}6 + {}^{-}2$

B. In Problem 2.2, you simplified chip boards to find the number represented. For example, each chip board below represents $^{+}3$.



Find three ways to represent $^{-}8$ on a chip board.

C. Jing-mei wants to find $^{-}8 - {}^{-}10$ by using a chip board. She puts eight red chips on the board to represent $^{-}8$ but then gets stuck because she cannot remove ten red chips to represent subtracting $^{-}10$.

How can Jing-mei show $^{-}8$ on a chip board so that she can remove ten red chips? What is $^{-}8 - {}^{-}10$? Explain how you determined your answer.

D. Drew wants to find $^{+}5 - {}^{+}7$, by using a chip board. How can he show $^{+}5$ on a chip board so that he can remove seven black chips to represent subtracting $^{+}7$? What is $^{+}5 - {}^{+}7$? Explain how you determined your answer.

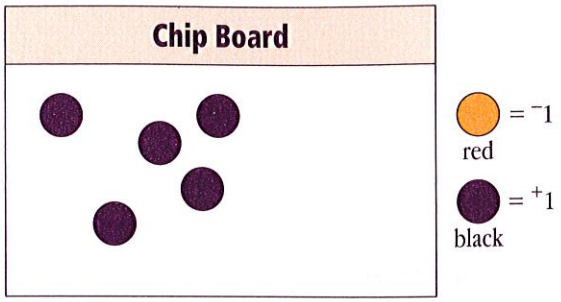
E. Use a chip board and black and red chips to find each difference. For each difference, tell how many chips of each color you used to represent the first integer so that you could take away chips to represent subtracting the second integer.

1. $10 - 12$ **2.** $7 - {}^{-}2$

3. $^{-}5 - 6$ **4.** $^{-}3 - {}^{-}7$

Problem 3.1 Follow-Up

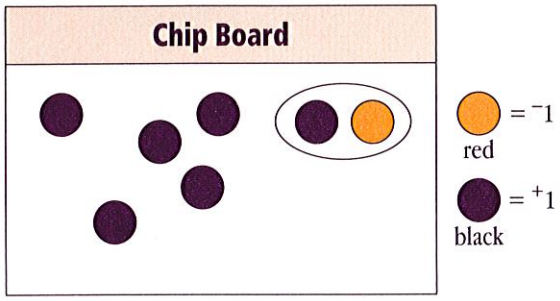
To find $^{+}5 - {}^{+}7$, Drew started by showing $^{+}5$ as five black chips.



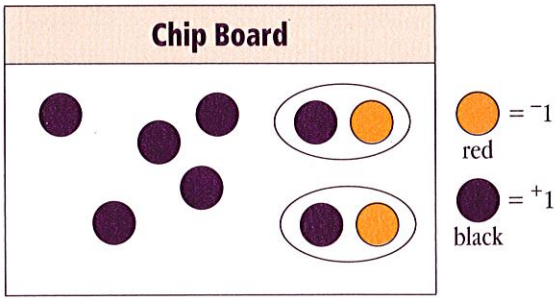
He could not represent subtracting $^{+}7$ because there were not seven black chips to remove from the board.

He recalled that adding or removing a black-red pair does not change the value of the board because such a pair represents 0 ($^{+}1$ and $^{-}1$ are opposites, so they combine to 0).

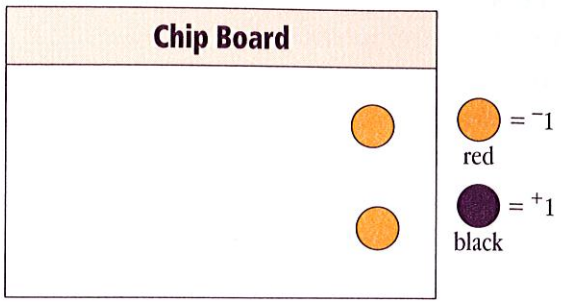
He added a black-red pair to the board.



The board now had six black chips and one red chip. To subtract $^{+}7$, Drew needed to remove seven black chips, so he added one more black-red pair.



Then Drew was able to represent the subtraction. He removed seven black chips from the board. Two red chips remained.



Drew wrote the number sentence $^{+}5 - {}^{+}7 = {}^{-}2$ to represent his work on the chip board.

1. Find three ways to show $^{-}5$ on the chip board. For each representation, write a subtraction problem that would be easy to solve if you started with that representation.

You have seen that there are lots of ways to represent a given integer on a chip board. For example, you could represent $^{+}5$ with eight black chips and three red chips or with six black chips and one red chip. However, there is only one way to represent a given integer with only one color. For example, the only way to represent $^{+}5$ with one color is by using five black chips, and the only way to represent $^{-}5$ with one color is by using five red chips.

The number of chips needed to represent an integer *with only one color* is the **absolute value** of the integer. Thus, the absolute value of 5 is 5, and the absolute value of $^{-}5$ is 5. We represent the absolute value of a number by writing a straight, vertical line segment on each side of the number. The equation $|{}^{-}5| = 5$ is read, "The absolute value of negative five equals five."

2. Find each absolute value.

a. $|{}^{-}7|$ **b.** $|18|$ **c.** $|{}^{-}42|$ **d.** $|0|$

3. Tell which numbers have the given number as their absolute value.

a. 12 **b.** 3 **c.** 31 **d.** 100