Magnificent Magnitude

2

Absolute Value

Lesson Overview

Students formalize the idea that opposites are the same distance from zero and call this distance the absolute value of a number. Students continually revisit the meaning of absolute value, focusing on distance from 0. Students evaluate absolute value statements and compare numbers using absolute values. Students solve problems using absolute value statements.

Grade 6 The Number System

Apply and extend previous understandings of numbers to the system of rational numbers.

7. Understand ordering and absolute value of rational numbers.
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
   d. Distinguish comparisons of absolute value from statements about order.

Essential Ideas

- The distance from zero is the absolute value, or magnitude, of a rational number.
- Absolute values are used to describe real-world situations.
- Absolute value equations are used to compute distance on a number line.

Lesson Structure and Pacing: 2 Days

Day 1

Engage

Getting Started: Going the Distance

Students use intuition to determine values that are 7 units from 0 on the number line.

Develop

Activity 2.1: Absolute Value as Magnitude

Students return to the human number line and plot values (opposites) that are the same distance from zero. Absolute value is defined and the notation is introduced. Students answer questions based on number lines to build their conceptual understanding of absolute value, including that the absolute value of 0 is 0.

Activity 2.2: Interpreting Absolute Value Statements

Students complete tables of situations, absolute value statements, and numeric values described by given and student-generated situations. The tables include statements of equality and inequality.

Day 2

Activity 2.3: Using Absolute Value to Solve Real-World Problems

Students complete problem-solving questions using absolute value expressions. They consider the distances traveled as workers travel up and down an elevator. Students consider gains and losses from a business balance sheet. They also analyze a situation involving positive and negative water levels. Students then compute the sum and difference of absolute value numbers and write absolute value statements representing the various situations.

Demonstrate

Talk the Talk: You Absolutely MUST Compare These!

Students use inequality symbols to compare absolute values and then justify their comparison.

Facilitation Notes

In this activity, students determine two values that are a given number of units from 0 on the number line.

Have students complete Questions 1 through 4 individually. Discuss as a class.

Questions to ask
- How is distance measured?
- What units might be associated with distance?
- Is distance always described using a positive number?
- Why can't negative numbers be used to represent distance?
- How do you determine distance on a number line?
- How are the number of intervals and the distance related to each other?
- How many directions from 0 are on a number line?

Misconception
When counting distance from one number to another on a number line, students will erroneously count tick lines rather than intervals.

Summary

There are exactly two locations the same distance from 0 on every number line.

Activity 2.1
Absolute Value as Magnitude

DEVELOP

Facilitation Notes

In this activity, the human number line is used to plot values and their opposites that are the same distance from zero. Absolute value is defined and the notation is introduced.

Note that the absolute value of 0 is 0.

Use masking tape to create a number line on the floor, include equally spaced tick marks and denote the location of 0. Ensure that the tick marks are spaced far enough apart that students can stand between the tick marks if you want to add other points. You will also need to cut 3 different lengths of string to be measured on the number line.

Select four students to participate in the activity and complete Question 1. As the class determines the possible locations for Students B, C, and D, the class should indicate the locations on their number lines.

Questions to ask
- How do the students know where to stand?
- What is true about the length of the string when the student moves from one side of 0 to the other on the number line?
- What does the string represent?
- How are the locations where Student B stood similar to each other? Student C? Student D?

Ask a student read the paragraph about absolute value aloud. Discuss the definition, focusing on absolute value of a rational number as the distance of the number from zero on a number line.

Differentiation strategy
To assist all students, have them highlight the definition; understanding absolute value represents distance is more meaningful than just memorizing "make a number positive."

Questions to ask
- Write |4| on the board and ask how to say this expression. What does it mean in terms of the number line?
- Write |−8| on the board and ask how to say this expression. What does it mean in terms of the number line?

Have students work with a partner or in groups to complete Questions 2–6. Share responses to as a class. Note that the focus is on the definition of absolute value. Refrain from using or accepting the phrase "absolute value is always positive or zero" as justification. Require students to reason about the number line.

As students work, look for
- Confusion between "opposite" and "absolute value".
- Haste to get the answer rather than making sense of the definition.

Questions to ask
- What is the distance from 5 to 0 on the number line?
- What is the distance from −7.2 to 0 on the number line?
- What is the distance from −5 to 0 on the number line? Is there another number that is the same distance from 0? If so, what is that number?
- What is the distance from $1\frac{5}{6}$ to 0 on the number line? Is there another number that is the same distance from 0? If so, what is that number?
- What is the distance from 0.75 to 0 on the number line? Is there another number that is the same distance from 0? If so, what is that number?

- What is the distance from −1.36 to 0 on the number line? Is there another number that is the same distance from 0? If so, what is that number?
- How does the number line help you see distance?
- How can you determine distance from zero on the number line?
- How can you use a number line to justify your answer?
- What is the definition of absolute value and how can that help you justify your answer?
- What happens to the absolute value of a number as that number's distance from zero on the number line increases?
- What happens to the absolute value of a number as that number's distance from zero on the number line decreases?
- Why is the absolute value of 0 equal to 0?
- Can you write a general rule for determining the absolute value of a number?

Summary

The absolute value of a rational number is number's distance from 0 on a number line.

Activity 2.2
Interpreting Absolute Value Statements

Facilitation Notes

In this activity, students complete tables of situations, absolute value statements, and numerical values described by given and student-generated situations.

Have a student read the paragraph and the first example in Question 1 aloud.

Questions to ask
- Did the temperature increase or decrease?
- How would you indicate an increase in temperature?
- How would you indicate a decrease in temperature?
- What is the *benchmark* or *zero* in thinking about temperature in this question?

Have students work with a partner or in groups to complete Question 1. Share responses as a class.

Questions to ask
- Did the account balance increase or decrease?

- What could it mean if the balance increased? Decreased?
- How do you indicate an increase? A decrease?
- Did the water level increase or decrease?
- What could it mean if the water level increased? Decreased?
- Did the elevation increase or decrease?
- What could it mean if the elevation increased? Decreased?
- In your situation, should your quantity increase or decrease?

Differentiation strategies
- If students work in groups of four, students can switch between shoulder and across-table partners from Question 1 to Question 2.
- For students who struggle, they could do fewer examples in each question but both questions are essential for meeting the standards in this activity.
- To extend the activity and focus on meaning, ask students to generate more situations.

Have a student read the paragraph and the first example in Question 2 aloud.

Questions to ask
- Draw a number line. How does the number line illustrate a water level less than $-2\frac{1}{2}$ feet?
- What language, or words, can indicate a negative water level?
- What other values could be used as examples for a water level less than $-2\frac{1}{2}$ feet?
- How would you discuss this situation in your everyday life? Would you use the statement in the Situation column or the Absolute Value Statement?

Have students work with a partner or in groups to complete Question 2. Note that this table is more difficult than Question 1. The goal in this question is for students to realize that they already use absolute value statements in everyday life. Share responses as a class.

Questions to ask
- Draw a number line for the situation and mark the given information.
- How does the number line illustrate the situation or the absolute value statement?
- What does it mean to have an account balance less than a negative dollar amount?
- What does it mean for your weight to change by a negative number of pounds?
- What is a height less than a negative number of feet?
- What does *colder than* mean?
- How do you indicate depth on a number line?
- Is depth a positive or negative number?

- In golf, the person with the lowest score wins. Par is the predetermined number of strokes that it should take to complete the tournament. Is below par good or bad?
- What is the benchmark, or 0, in the golf tournament situation?

As students share their responses, students should critique each other's situations, absolute value statements, and numerical examples.

Questions to ask
- Can you provide additional examples?
- On the number line, where are all of the examples for the situation? Can you write an inequality statement for where to locate the possible examples?
- How would you discuss this situation in your everyday life? Would you use the statement in the Situation column or the Absolute Value Statement?

Summary

Absolute value statements are used in everyday life to discuss relationships between numbers, particularly negative numbers or ideas that are expressed with words that denote a negative direction, e.g., depth, debt, below.

Activity 2.3
Using Absolute Value to Solve Real-World Problems

Facilitation Notes

In this activity, students use absolute value expressions to answer questions in everyday life scenarios. They compute the sum and difference of absolute value numbers and write absolute value statements representing the various situations.

Have a student read Question 1 aloud. Discuss as a class. You may want to consider showing students pictures of elevator panels that display separate ground level and first floors for students, pictures of elevator panels that include negative numbers, and/or pictures of garages labeled with negative floors.

Questions to ask
- Have you ever seen an elevator with a different ground floor and first floor?
- Have you ever seen negative numbers on an elevator panel?
- How is this building plan like a number line?

Complete Question 1, part (a) as a class. All students should read the first part of the Who's Correct and consider how they would determine the distance traveled. Then they should read the solutions proposed by Taylor and Cecelia.

Questions to ask
- Why would Taylor think to subtract 9 and 3?
- Cecelia wrote an absolute value equation. Why did she use absolute values? How did she know to add the absolute values?
- Use the meaning of absolute value to explain why Cecelia's strategy makes sense.
- How would the answer be different if the ground floor was also the first floor?

Have students work with a partner or in groups to complete the remainder of Question 1. Share responses as a class.

Differentiation strategy
For students who struggle, provide a template with 3 copies of the building and 3 number lines to support their work.

Questions to ask
- Did you draw a number line?
- How does the number line help you answer these questions?
- Which floor of the office building is associated with a value of zero?
- Which floors of the office building are associated with positive numbers?
- Which floors of the office building are associated with negative numbers?
- How does the concept of absolute value apply to this situation?
- If you traveled up 2 floors and then traveled down 3 floors, what does the absolute value of this situation tell you?
- How do you determine whether to use addition or subtraction when writing a numeric expression containing absolute values?

Have students work with a partner or in groups to complete Questions 2–3. Share responses as a class.

Differentiation strategy
Place students in groups of four. Have half of the groups complete Question 1 and half of the groups complete Question 2. Afterwards, regroup students so that each group has 2 members that completed each problem. Have each pair lead the other pair is solving the question that they previously solved.

Misconception
When solving estimation problems, such as Question 2, part (a), be sure to check the process as well as the result. Sometimes students calculate

the response, then round their answer. Explain the estimation process of rounding first, then calculating using the rounded figures for a quick calculation.

Questions to ask for Question 2
- Which week did the company make $175?
- Which week did the company owe $12.05?
- How much money did the company owe at the end of week 2?
- How did you estimate the gain and loss between consecutive weeks? What operation(s) did you use?
- How did you determine between which two weeks the company experienced the largest gain in money?
- How did you determine between which two weeks the company experienced the largest loss in money?
- How did you determine the week when the company had the highest balance? The lowest balance?
- How can you use absolute value equations to answer these questions? How do you determine whether to use addition or subtraction when writing a numeric expression containing absolute values?
- What does the comparison of the absolute values of the gains and losses tell you about the company's success?
- Is the company doing well? Explain.

Questions to ask for Question 3
- Did the water level in the pond increase or decrease from week 1 to week 2? Explain.
- How much did the water level change from week 3 to week 4?
- How did you determine between which two weeks the water level changed the most?
- How did you determine between which two weeks the water level changed the least?
- How can you use absolute value equations to answer these questions? How do you determine whether to use addition or subtraction when writing a numeric expression containing absolute values?
- Overall, is the depth of the pond decreasing or increasing? Explain.

Questions to ask during class discussion
- How are Questions 2 and 3 similar? How are they different?
- What common strategies did you use to answer the questions?
- What are some other contexts in which you could use absolute value equations to solve problems?

Summary

Absolute value equations can be used to determine distances between positive and negative numbers in a variety of contexts.

Talk the Talk: You Absolutely MUST Compare These!

Facilitation Notes

In this activity, students use symbols ($<$, $>$, or $=$) to compare absolute values and use the definition of absolute value to justify their comparison.

Have students complete this activity individually. Ask them to share and compare with a partner. Encourage students to draw number lines to support their comparisons.

Differentiation strategy

If time, act out some or all of these comparisons on the Human Number Line. For example, ask a student to stand at 0. Ask another student to stand at approximately -4.67 and unroll a length of string between the two students. Ask a third student to stand at 3 and unroll a length of string between the student at 0 and the student at 3. Compare the lengths of the strings. The longer string has the greater absolute value because absolute value is distance from zero.

Questions to ask

- Why is $|-4.67|$ greater than $|3|$ when -4.67 is less than 3?
- Why is $\left|25\frac{9}{10}\right|$ less than $\left|-33\frac{2}{3}\right|$ when $25\frac{9}{10}$ is greater than $-33\frac{2}{3}$?
- Why is $|13.45|$ less than $|-27|$ when 13.45 is greater than -27?
- When is the absolute value of a negative number greater than the absolute value of a positive number?
- When is the absolute value of a positive number greater than the absolute value of a negative number?

Summary

Absolute values can be compared using equality and inequality symbols.
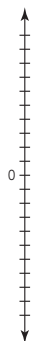
Magnificent Magnitude

2

Absolute Value

Warm Up Answers

1.

2.

3.

In each question, the numbers are on opposite sides of 0 and are the same distance from 0.

WARM UP

Plot each set of numbers on the number line and describe the relationship between the numbers.

1. 5 and −5
2. $2\frac{3}{4}$ and $-2\frac{3}{4}$
3. 8.634 and −8.634

LEARNING GOALS

- Explain the meaning of the absolute value of a rational number as its distance from 0 on a number line.
- Interpret the meaning of absolute value as the magnitude for a positive or negative quantity in a real-world context.
- Evaluate the absolute value of a quantity.
- Compare and order numbers expressed as absolute value and distinguish absolute value comparisons from statements about order.
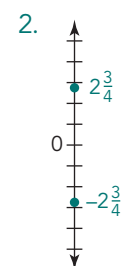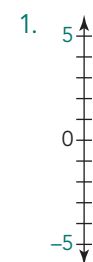
KEY TERM

- absolute value

Numbers can be described by their distance from 0 on the number line. How can you use these distances to solve real-world problems?

Answers

1. See below.

2. The distance is 7 units.

3. See below.

4. There are 2 numbers that are exactly 7 units from 0 -- one in the positive direction and one in the negative direction.

Going the Distance
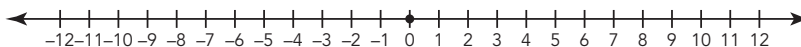
1. Plot a point at −7 on the number line.

2. Describe the distance from −7 to 0.

3. Plot as many other points as possible on the number line that are the same distance from 0 as −7.

4. How many numbers did you plot? Why do you think this is true?

1a. The distance from 0 was the same each time a specific student changed positions.

1b. The approximate values were the positive and negative of the same number.

ACTIVITY 2.1

Absolute Value as Magnitude

Let's revisit the number line from the *Human Number Line* lesson.

Your teacher will assign students to participate in the activity.
Be sure to record what happens on the number line.

- Student A: Stand on 0 and hold one end of the string provided by your teacher.

- Student B: Hold the other end of the string and stand on the number line as far as possible from Student A. Are there other places on the number line that you could stand and be as far from Student A as possible?

- Repeat this activity with two more pieces of string of different lengths and two additional students, Students C and D. Student A will hold the 0 end of each string.
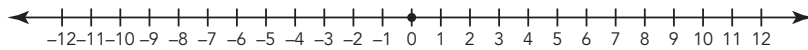
1. Compare the locations where each student stood.

   a. What do you notice about the distances each time the students moved?

   b. What do you notice about the approximate values for the numbers where each stood?

The magnitude, or **absolute value,** of a number is its distance from zero on a number line. The symbol for absolute value is | |. The expression |*n*| is read as "the absolute value of a number *n*."
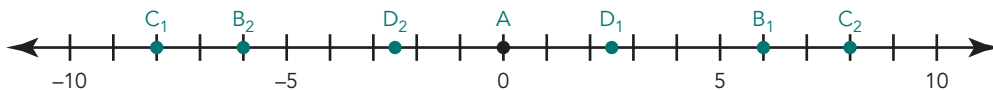
Because distance cannot be negative, the absolute value of a number is always positive or 0.
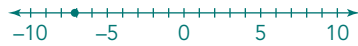
Answers

2. See below for sample answer.

2a. 5 is 5 units from 0.

2b. $|5| = 5$

3. Sample number line:

3a. -7.2 is a little more than 7 units, or 7.2 units, from 0.

3b. $|-7.2| = 7.2$

4. Sample answers.

4a. -5 is 5 units from 0 on the number line. 5 is also 5 units from the zero on the number line.

4b. $1\frac{5}{6}$ is $1\frac{5}{6}$ units from 0 on the number line. $-1\frac{5}{6}$ is also $1\frac{5}{6}$ units from zero on the number line.

4c. 0.75 is three-quarters of a unit from zero on the number line. -0.75 $\left(-\frac{3}{4}\right)$ is also three-quarters of a unit from zero on the number line.

4d. -1.36 is 1.36 units from zero on a number line. 1.36 is also 1.36 units from zero on the number line.
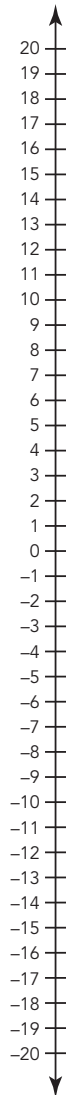
5. Except in the case of 0, there are always two numbers that have the same absolute value.

Sample answers.

6a. The absolute value of any positive number is the same as the number.

6b. The absolute value of any negative number is its opposite.

6c. The absolute value of 0 is 0.

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2. **Plot 5 on the number line.**

   a. How far is 5 from 0?   b. $|5| =$ _____

3. **Plot −7.2 on the number line.**

   a. How far is −7.2 from 0?   b. $|-7.2| =$ _____

4. **Explain what each statement means. Name any other values that have the same absolute value, if possible.**

   a. $|-5|$ b. $|1\frac{5}{6}|$

   c. $|0.75|$ d. $|-1.36|$

Use your investigation and a number line to answer each question.

5. **Can two different numbers have the same absolute value? If so, provide examples.**

6. **What can you say about the absolute value of**

   a. any positive number?

   b. any negative number?

   c. zero?

2.

ACTIVITY 2.2 Interpreting Absolute Value Statements

Absolute values are used in real-world applications when you are interested in only the number and not in the sign of the number. When you look at temperature changes, you could say the temperature "fell by," "decreased by," or "increased by" an absolute value.

1. **Complete the table with an appropriate situation, absolute value statement, and/or number. For the last row, assign the correct units to the number based on your situation.**

Situation	Absolute Value Statement	Numeric Example (with units)
The temperature went from 55°F to 5°F.	The temperature fell by 50°F.	−50°F
The bank account balance went from $2500 to $2250.		
The bank account balance went from $495 to $615.		$120
	The water level increased by 4.9 feet.	
During the hike, the elevation went from 1125 feet to 1750 feet.		
		−10_____

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Answers

1.

Situation	Absolute Value Statement	Numerical Example (with units)
	He spent $250.	−$250
	He earned $120.	
The water level went from 2 feet to 6.9 feet.		4.9 feet
	During the hike, the elevation increased by 625 feet.	625 feet
Sample answer. The water level went from 25 feet to 15 feet.	Sample answer. The water level decreased by 10 feet.	feet

ELL Strategies

Have students use the examples shown in the tables to create stories about absolute value. English Language Learners can choose a role from the list for a storytelling circle: discussion facilitator, passage picker, illustrator, connector, summarizer, vocabulary enricher, and investigator. Give students sentence starters and a wordlist to facilitate the discussion.

2.

Situation	Absolute Value Statement	Numerical Example (with units)
		−$35
		−8 lbs
	Deeper than 350 feet below the water	−400 feet
A temperature less than −10°F		−15°F
A level less than −15 m		−16 m
A golf stroke total less than −7 strokes off par		−8 strokes
An account balance less than −$95	A debt greater than $95	dollars

You also use absolute value statements to describe how numbers compare with other numbers. You often use these statements without thinking about "less than" or "greater than." Rather, you use words like "debt," "lost," "colder," "depth," "above," "hotter," or "below."

2. **Complete the table with an appropriate situation, absolute value statement, and/or example. For the last row, assign the correct units to the numeric example based on your situation.**

Situation	Absolute Value Statement	Numeric Example (with units)
A water level less than $-2\frac{1}{2}$ feet	More than $2\frac{1}{2}$ feet below a full pool	−3 feet
An account balance less than −$30	A debt greater than $30	
A weight less than −7.5 pounds of previous weight	Lost more than 7.5 pounds	
A dive to a height less than −350 feet		
	Colder than 10 degrees below 0	
	A depth greater than 15 m	
	A golf tournament stroke total more than 7 strokes below par	
		−100_____

Par is the number of strokes, or swings, a golfer is expected to take.

Sample answers.

1a. Cecelia is correct. To determine how far Melanie travels, they must add the distances from the garage to the ground floor and from the ground floor to the office floor.

ACTIVITY 2.3

Using Absolute Value to Solve Real-World Problems

1. In many buildings, particularly outside of the United States, the ground floor of a building is labeled as G or Lobby. The first floor of the building is one floor above the ground floor. The building pictured has a lobby, 10 floors of offices, and 4 floors of garage below the lobby.
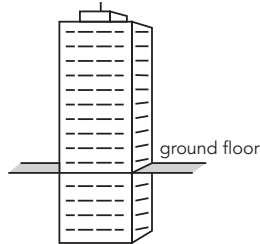
ground floor

a. Melanie has an office on the 9th floor and parks on the 3rd floor below the ground floor. Taylor and Cecelia are determining how many floors Melanie must go up from her car to reach her office.

Taylor represents the 9th floor as 9 and the 3rd floor below ground as −3. Therefore, since 9 − 3 = 6, Melanie traveled 6 floors to get from her car to her office.

Cecelia says that the ground floor to the 9th floor is 9 floors, and from the ground floor to the 3rd garage level is 3 floors. Melanie traveled |9| + |−3| = 9 + 3 = 12 floors.

Who is correct? Explain your reasoning.

ELL Strategies

Work with students to write words signifying direction: *horizontal*, *vertical*, *up*, *down*, *right*, *left*, etc. Have English Language Learners draw a mathematical operator and/or a picture they associate with the word. For example, for the word *down*, students may write an arrow pointing downward with a subtraction symbol. Tell students that some words, such as *vertical*, may have more than one operator associated with them.

Answers

1b. $|-2| + |7| = 2 + 7 =$
    9 floors

1c. $|8| + |-4| + |4| = 8 + 4 + 4 = 16$ floors

1d. $|10| - |5| = 10 - 5 = 5$ floors

Sample answers.

2a. Week 1 to 2 ≈ $360 loss
    Week 2 to 3 ≈ $430 gain
    Week 3 to 4 ≈ $690 loss
    Week 4 to 5 ≈ $300 gain
    Week 5 to 6 ≈ $150 gain
    Week 6 to 7 ≈ $290 gain
    Week 7 to 8 ≈ $100 loss
    Week 8 to 9 ≈ $15 gain
    Week 9 to 10 ≈ $20 gain

2b. Top Notch had the greatest gain between weeks 2 and 3. The actual gain was $|-201.35| + |231.57| = 201.35 + 231.57 = \$432.92$.

2c. Top Notch had the greatest loss between weeks 3 and 4. The actual loss was $|231.57| + |-456.45| = 231.57 + 456.45 = \$688.02$.

2d. The difference between the lowest and highest balance, weeks 4 and 7, was $|-456.45| + |281.34| = \$737.79$.

Write a numeric expression using absolute values that would represent each situation. Then calculate the answer.

 b. Caleb parks his car on the 2nd floor below ground and works on the 7th floor. How many floors must he go up from his car to reach his office?

 c. Lucinda is working on the 8th floor. At lunch, she goes to her car on the 4th floor below ground, and then back up to the lobby. How many total floors does Lucinda travel?

 d. If Damon goes from his office on the 10th floor to a meeting on the 5th floor, how many floors does he travel and in which direction?

2. The Top Notch company's bank balances are shown. The table represents the first 10 weeks of operation. Overdrafts are represented by amounts within parentheses.

Week	1	2	3	4	5	6	7	8	9	10
Balance	$159.25	($201.35)	$231.57	($456.45)	($156)	($12.05)	$281.34	$175	$192.34	$213

 a. Use estimation to determine the gains/losses between consecutive weeks.

 b. Between which two weeks did Top Notch have the largest gain in money? What was the actual gain?

 c. Between which two weeks did Top Notch have the largest loss in money? What was the actual loss?

 d. What was the difference between the company's lowest balance and its highest balance?

e. Order the estimated gains and losses that you determined in part (a) from least to greatest. Use a negative sign to indicate losses.

f. Order the estimated gains and losses that you determined in part (a) from least to greatest according to their absolute values. What does the absolute value mean in the context of this problem?

g. Why are the orders different in parts (e) and (f)?

3. As part of a long-term science experiment, two rulers were connected at zero and used to measure the water level in a pond. The connected rulers were placed in the pond so that the water level aligned at zero. The water level was measured each week for 10 weeks.

Week	1	2	3	4	5	6	7	8	9	10
Water level	$2\frac{3}{4}$	$-2\frac{1}{8}$	$1\frac{7}{8}$	$-\frac{3}{4}$	$\frac{3}{4}$	$1\frac{1}{8}$	$-\frac{7}{8}$	$1\frac{1}{4}$	-2	$-\frac{3}{16}$

a. What do the positive numbers represent? What do the negative numbers represent?

b. Between which two weeks did the water level change the most? What was the change?

c. Between which two weeks did the water level change the least? What was the change?

d. How much did the water level change between Weeks 4 and 5? What was the change?

Answers

Sample answers.

2e. −$690, −$360, −$100, $15, $20, $150, $290, $300, $430

2f. |15|, |20|, |−100|, |150|, |290|, |300|, |−360|, |430|, |−690|. These values indicate how far from the break-even point, or no gain or loss, on the week's balance sheet.

2g. The first ordering is based on positive and negative values. The second ordering is based on distance from zero.

Sample answers.

3a. The positive values represent water levels above the water level as measured at week zero; these values are read from the top ruler. The negative numbers represent water levels below the water level as measured at week zero; these values are read from the bottom ruler.

3b. The water level changed the most between weeks 1 and 2. $|2\frac{3}{4}| + |-2\frac{1}{8}| = 2\frac{3}{4} + 2\frac{1}{8} = 4\frac{7}{8}$ inches

3c. The water level changed the least between weeks 5 and 6.

$|1\frac{1}{8}| - |\frac{3}{4}| = 1\frac{1}{8} - \frac{3}{4} = \frac{3}{8}$ inches

3d. $|-\frac{3}{4}| + |\frac{3}{4}| = 1\frac{1}{2}$. The water level went up $1\frac{1}{2}$ inches.

NOTES

TALK the TALK

You Absolutely MUST Compare These!

Insert a $>$, $<$, or $=$ symbol to make each statement true. Justify each answer in terms of the definition of absolute value and number lines.

1. $|-4.67| \_\_ |3|$

2. $|-15| \_\_ |15|$

3. $|25\frac{9}{10}| \_\_ |-33\frac{2}{3}|$

4. $|13.45| \_\_ |-27|$

5. $|-15.34| \_\_ |-1\frac{11}{12}|$

6. $|-19\frac{1}{2}| \_\_ |5.5|$

Assignment

Write

Explain the relationship between a number, its opposite, and its absolute value.

Remember

The absolute value of a rational number is its distance from zero on a number line. Absolute value equations can be used to calculate the distance between positive and negative numbers.

Practice

1. Julio is a wrestler for his high school wrestling team in the winter. Julio needs to stay around 140 pounds in the off-season. He charted his weight over the summer by listing the differences his weight was from 140 pounds. He uses negative numbers when his weight was under 140 pounds and positive numbers when his weight was above 140 pounds.

Week	1	2	3	4	5	6	7	8	9	10	11	12
Weight Difference	+4.5	+2.1	−1.5	−0.5	−2.5	+1.5	−3.75	−2.8	0	+1.3	−1.5	−5

a. Was the amount his weight varied from 140 pounds in week 4 more or less than the amount it varied from 140 pounds in week 8?
Insert a $>$, $<$, or $=$ symbol to make the statement true. Explain your answer.
$$|-0.5| \bigcirc |-2.8|$$

b. Was the amount his weight varied from 140 pounds in week 6 more or less than the amount it varied from 140 pounds in week 11?
Insert a $>$, $<$, or $=$ symbol to make the statement true. Explain your answer.
$$|+1.5| \bigcirc |-1.5|$$

c. Use absolute values to determine the difference in Julio's weight from week 7 to week 10.

d. Use absolute values to determine the difference in Julio's weight from week 8 to week 12.

2. The table shown tracks Julio's weight changes that he reports to his coach for the first 4 weeks of school. Complete the table to explain the changes.

Situation	Absolute Value Statement	Rational Number
His weight went from 140 to 135 pounds.	His weight fell by 5 pounds.	
His weight went from 135 pounds to 141 pounds.		6 lb
His weight went from 141 pounds to 140.5 pounds.		
His weight went from 140.5 pounds to 139 pounds.		

Assignment Answers

Write

Answers will vary.

Practice

1a. $<$

The weight variation in week 4 was less than the weight variation in week 8. In week 4 the amount of variation was 0.5 pounds, while in week 8 the amount of variation was 2.8 pounds.

1b. $=$

The weight variation in week 6 was equal to the weight variation in week 11. In week 6 the amount of variation was 1.5 pounds, while in week 11 the amount of variation was also 1.5 pounds.

1c. $|1.3| + |-3.75| = 5.05$

1d. $|-5| - |-2.8| = 2.2$

2.

Situation	Absolute Value Statement	Rational Number
		−5 lb
	His weight increased by 6 pounds.	
	His weight fell by 0.5 pounds.	−0.5 lb
	His weight fell by 1.5 pounds.	−1.5 lb

3a. $|-33| + |50| = 33 + 50$
$= 83$

The temperature rose 83 degrees.

3b. $|-5| + |37| = 5 + 37$
$= 42$

The temperature rose 42 degrees.

3c. $|-4| + |45| = 4 + 45$
$= 49$

The temperature rose 49 degrees.

4a. 4 days

4b. It rained the most between days 7 and 8.

$|-2.4| + |5.6| = 2.4 + 5.6$
$= 8$

The gauge recorded 8 inches of rain.

4c. Evaporation was the greatest between days 5 and 6.

$|2.1| + |0.9| = 2.1 + 0.9$
$= 3$

The gauge recorded evaporation of 3 inches of water.

4d. $|0.5| + |-1.3| = 0.5 + 1.3$
$= 1.8$

The change in the water level in the gauge was -1.8 inches.

4e. $|-1.3| + |3.7| = 1.3 + 3.7$
$= 5$

The change in the water level in the gauge was $+5$ inches.

Stretch

1. Sample answer: At the end of the tournament, the golfer was 12 strokes under par.

2. Sample answer: The student lost $4\frac{1}{2}$ points on the project.

3. Sample answer: The student deposited $7.30 into the bank.

4. Sample answer: The dieter lost 0.7 pounds last week.

3. Weather experts collect many types of data to study and analyze, including extreme temperature changes. The interior West of North America experiences great temperature changes due to Chinook Winds. The table shows extreme temperature rises in three cities.

Place	Granville, ND	Fort Assiniboine, MT	Spearfish, SD
Date	Feb. 21, 1918	Jan. 19, 1892	Jan. 22, 1943
Time Period	12 hours	15 minutes	2 minutes
Temperature Change	From −33°F to 50°F	From −5°F to 37°F	From −4°F to 45°F

For each city, write an absolute value equation and use it to determine how much the temperature rose.

a. Granville, ND   b. Fort Assiniboine, MT   c. Spearfish, SD

4. Tyler measured the rainfall and evaporation using a rain gauge in his backyard for 8 days. Tyler marked his rain gauge with values from −6 inches to +6 inches and filled the gauge with water to the zero mark. For each question, write an expression using absolute value and then calculate the answer.

Days	1	2	3	4	5	6	7	8
Gauge Reading	0.5	−1.3	3.7	4.2	2.1	−0.9	−2.4	5.6

a. On how many days out of the eight did it rain?

b. Between which two consecutive readings did it rain the most? How many inches were recorded?

c. Between which two consecutive readings was evaporation the greatest? How many inches of water evaporated?

d. Calculate the gain or loss of water in the rain gauge between days 1 and 2. Express the change in the water level in the gauge as a positive or negative number.

e. Calculate the gain or loss of water in the rain gauge between days 2 and 3. Express the change in the water level in the gauge as a positive or negative number.

Stretch

Write a scenario to represent each rational number.

1. −12   2. $-4\frac{1}{2}$ 3. 7.3   4. −0.7

Review

1. Use the $>$, $<$, or $=$ symbol to complete each statement.

a. $-5 \bigcirc -8$ b. $-3 \bigcirc 0$ c. $5 \bigcirc -5$

2. Five employees work on the receiving dock at a factory. They divide the number of crates they unload from each truck equally. Define variables for the number of crates on a truck and for the number of crates each employee unloads from the truck. Write an equation that models the relationship between these variables.

3. Solve for the variable in each equation.

a. $\frac{t}{2} = 15$ b. $y - 8 = 19$

Review

1a. $>$

1b. $<$

1c. $>$

2. Let t represent the number of crates on a truck and let c represent the number of crates each employee unloads from the truck.

$c = \frac{t}{5}$

3a. $t = 30$

3b. $y = 27$