3

There Are Many Paths ...

Problem Solving on the Coordinate Plane

WARM UP

Solve each equation.

1. $120 + h = 315$
2. $w - 17 = 38$
3. $\frac{c}{5} = 12$
4. $169 = 13w$

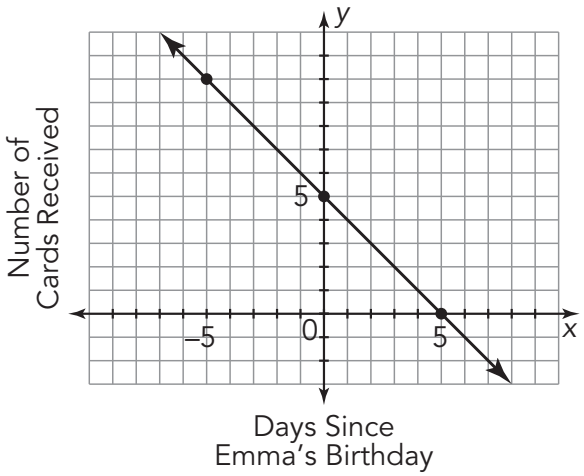
LEARNING GOALS

- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.
- Interpret the meaning of points plotted on the coordinate plane.
- Use equations to solve real-world problems.
- Use graphs relating an independent and dependent quantity changing in relationship to one another to solve real-world problems.
- List advantages and disadvantages of different representations for solving real-world and mathematical problems on the coordinate plane.

Now that you understand how to plot points in all four quadrants of the coordinate plane, you can solve many more types of problems than you could previously. How can you use graphs and equations to solve problems?

Emma's Birthday

Analyze the graph.



1. **Explain what you can determine about the situation from the graph.**

2. **What do the plotted points mean in terms of this situation?**

3. **Do all of the values on the line make sense in terms of the situation?**

4. **Can you determine an equation for the graph?**

ACTIVITY
3.1 Weigh In

Julio is a wrestler for his high school team. Although he does not wrestle during the 12 weeks of summer, his coach would like him to stay around 140 pounds so that he doesn't have to work so hard during the season to stay in his 142-pound weight class. Julio charted his weight over the summer.

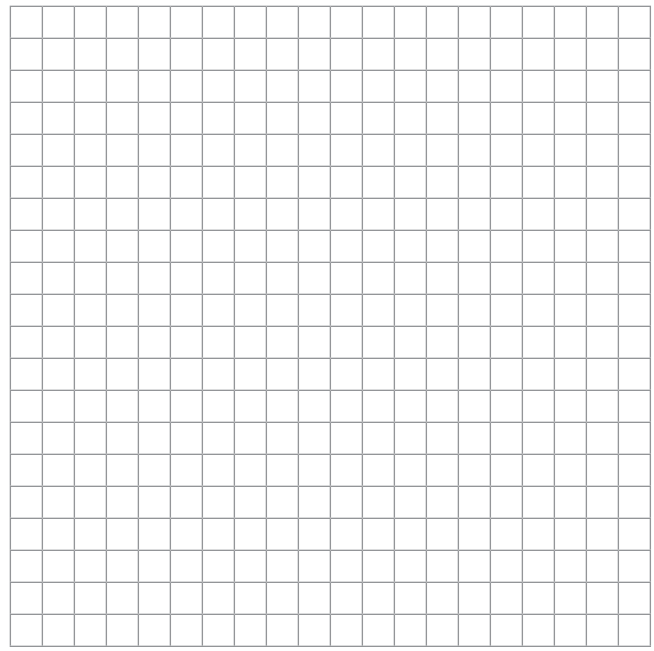
Week	1	2	3	4	5	6	7	8	9	10	11	12
Weight	144.5	142.1	138.5	139.5	137.5	141.5	136.25	137.2	140	141.3	138.5	135
Weight Differential												

1. Consider the table shown.

a. Which quantity is the independent quantity and which is the dependent? Explain your reasoning.

b. What is the unit for each quantity?

c. Which quadrant(s) will you need in order to plot Julio's data? Draw and label your axes. Then graph the data.

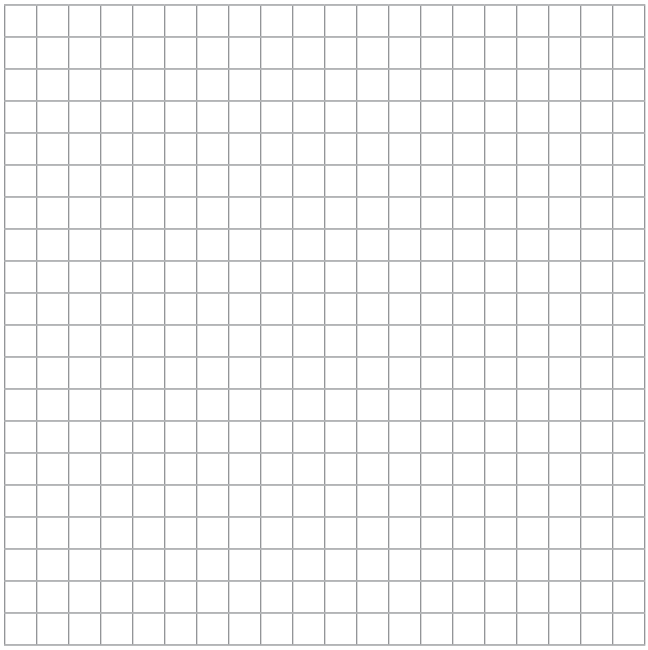


2. **The coach was impressed with Julio's data collection, but he was interested in how much Julio's weight varied from 140 pounds each week.**

   a. **Complete the last row of weight differentials, the differences of Julio's weight from 140 pounds. Use negative numbers when the weight is below 140 pounds and positive numbers when his weight is above 140 pounds.**

   b. **What is the dependent quantity in this situation?**

   c. **Which quadrant(s) will you need in order to plot Julio's data for the coach's request? Draw and label your axes, including the units. Then graph the data.**



3. Compare the two approaches taken by Julio and his coach.

   a. Compare the independent and dependent quantities.

   b. Compare the graphs. What do you notice about the patterns of the points?

   c. Explain the meaning of the *x*-axis in each approach.

   d. Why do you think the coach preferred his approach over Julio's approach?

4. Use the table and graphs to answer each question.

   a. Between which two consecutive weeks did Julio's weight change the most? What was the weight change?

   b. What is the difference between Julio's highest weight and his lowest weight?

   c. Which representation—table, Julio's graph, the coach's graph—did you use to answer the questions? Why did you make those choices?

   d. If you were Julio's coach, what advice would you give Julio?

ACTIVITY
3.2 An Interesting Day in South Dakota

An interesting day of temperature changes occurred in Rapid City, South Dakota, on January 22, 1943. The table shows the temperature changes that happened throughout the day.

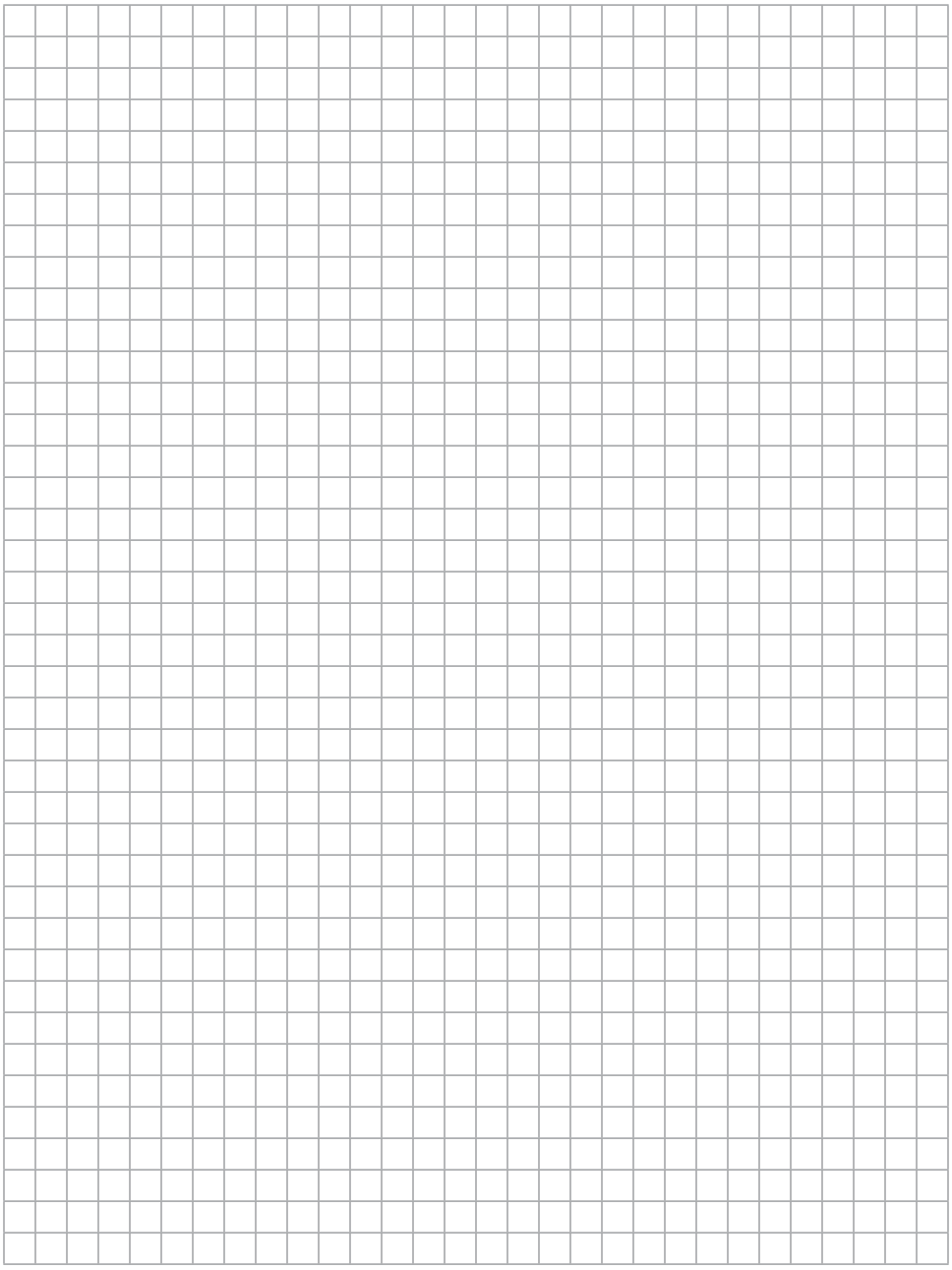
Time	Temperature (°C)
10:30 A.M.	−6.7
10:35 A.M.	13.3
12:00 P.M.	15.6
12:05 P.M.	−10.6
12:35 P.M.	−9.4
12:40 P.M.	10
2:20 P.M.	14.4
2:25 P.M.	−8.3

Create a graph of the temperature changes.

1. Which quadrants do you need for your graph? Explain your reasoning.

2. Draw and label the axes for the graph. Then graph the data and connect consecutive points.

NOTES



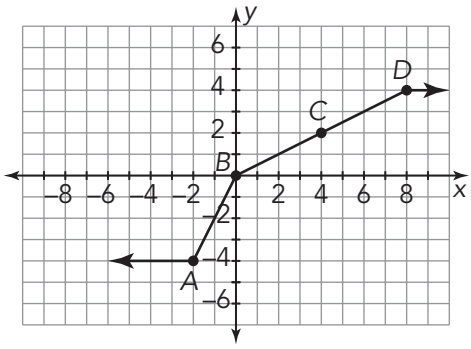
3. Between which two times was the temperature swing the greatest?

4. Describe the pattern. Why is this called an "interesting" day?

ACTIVITY
3.3 No Place Like Home

Suppose this graph summarizes your day. The *x*-axis of this graph represents time in minutes from 12:00 P.M., and the *y*-axis represents your distance from home in blocks. Locations north of your house are positive, and locations south of your house are negative. A point at the origin represents you being home at 12:00 P.M.

Graph A

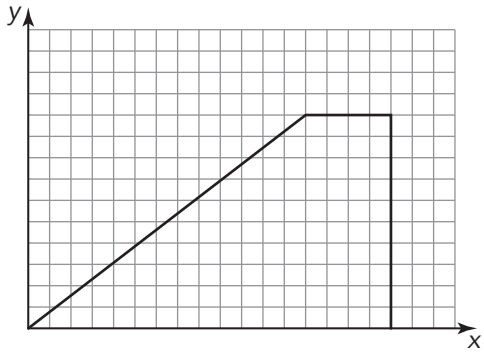


1. Describe the meaning of each of the four labeled points.

x	*y*	Meaning
−2	−4	
0	0	
4	2	
8	4	

2. Adrian and Sierra are discussing how the graph should look before $x = -6$ and after $x = 10$. Adrian thinks he should draw arrows to indicate that the graph continues to the left and right, respectively. Sierra disagrees and thinks they should draw segments back to the *x*-axis. Who is correct?

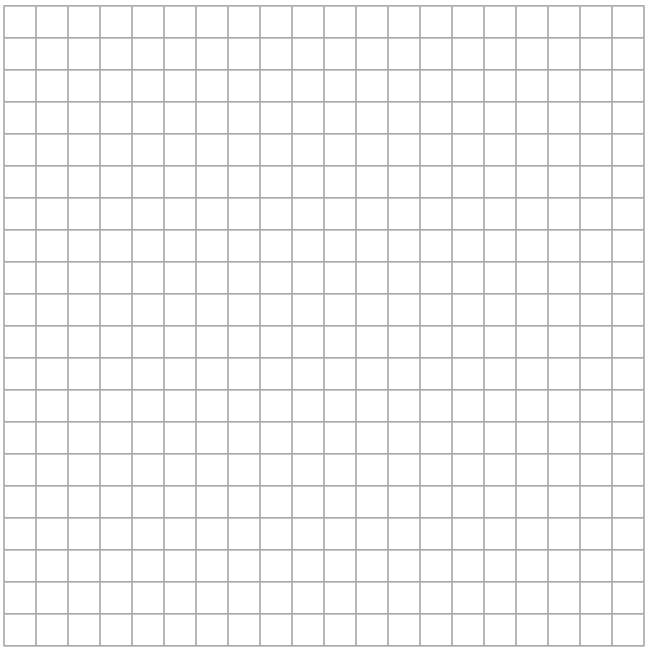
Let's consider another graph.



3. Write a possible scenario for this graph. Be sure to specify the units and the meaning of the origin for your scenario.

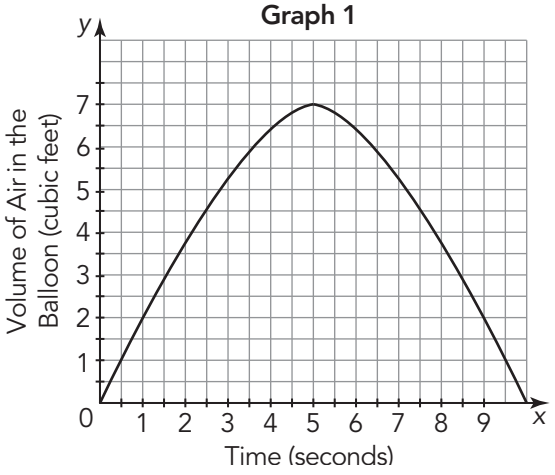
Natasha and her family took a 3-day trip to her grandmother's house. On the first day, they drove 300 miles. On the second day, they drove 350 miles. On the third day, they drove the remaining 200 miles.

4. Create a graph to represent Natasha's family trip. Be sure to label your axes with quantities and units and label specific points that highlight the trip.

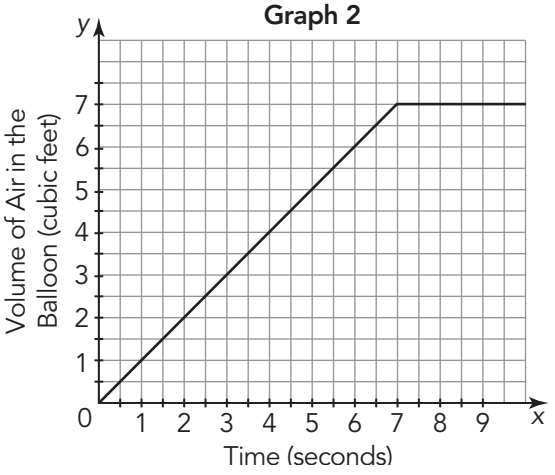


Nadja is coordinating the neighborhood Spring Fling. She asks Matthew to blow up balloons for the event. The graphs shown represent his efforts.

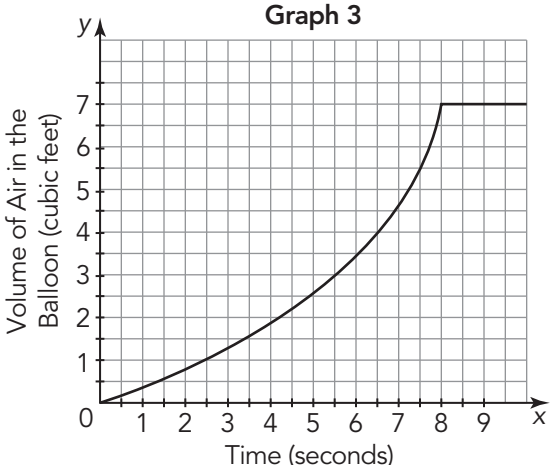
Graph 1



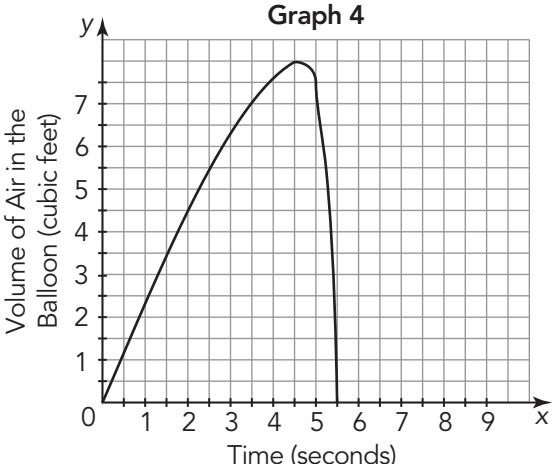
Graph 2



Graph 3



Graph 4



5. Analyze each graph shown, and then answer each question.

a. What quantity is represented on the *x*-axis in each graph?

b. What quantity is represented on the *y*-axis in each graph?

c. Which quantity is independent quantity and which is dependent quantity?

6. Match each description with the appropriate graph.

   a. Matthew blows air into a balloon at a steady rate, then ties it off when it is full.

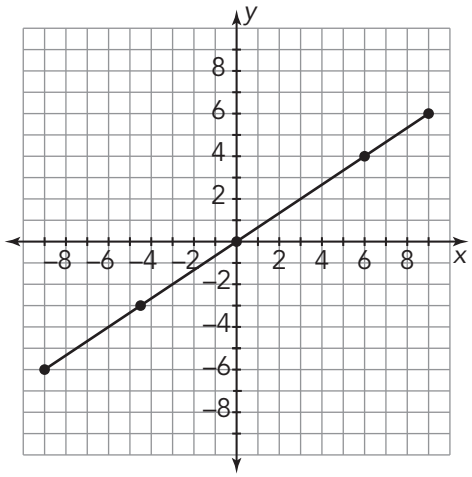
   b. Matthew blows air into a balloon, and then the balloon pops!

   c. Matthew blows air into a balloon, and then lets the air out.

   d. Matthew blows air into a balloon slowly. As the balloon stretches out, he is able to blow more air into the balloon. He then ties off the balloon when it is full.

ACTIVITY
3.4 Pool Level

The graph shows the water level of a pool. The *x*-axis represents time, in hours, and the *y*-axis represents the water level, in inches. The origin represents 3:00 P.M. and the desired water level.



1. Label the graph with the independent and dependent quantities and their units.

2. Create a table of values for the points plotted and describe the meaning of each.

x	*y*	Meaning

3. At what rate did the water go into the pool? Explain your reasoning.

4. Describe a situation that would match the graph.

5. Write an equation for this situation.

6. Why does the graph stop rather than continue infinitely?

7. Using any of your mathematical tools, determine the time when the pool was 3 inches above the desired fill level. Is your answer exact or approximate? Explain.

NOTES

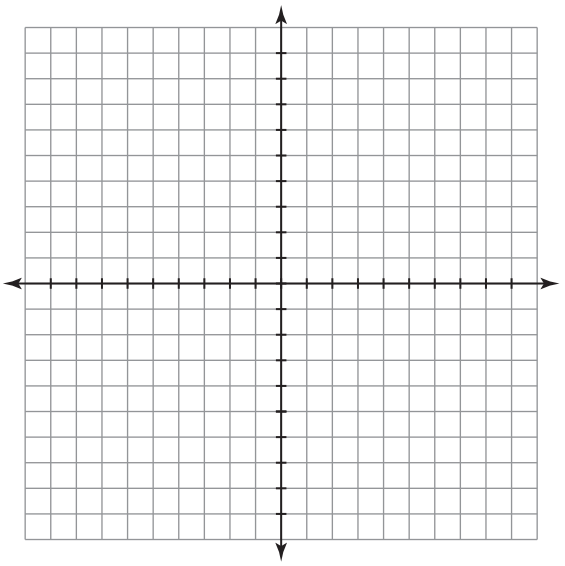
ACTIVITY 3.5

Water in the Bucket

As part of a science project, Damon collected water in a bucket in his backyard and is studying the evaporation. Unfortunately, Damon is a bit forgetful and forgets to take measurements of the water every day. The first day he remembered was Sunday, which was 4 days AFTER the data collection was to begin. He collects the following data.

Days Since Sunday	Height of Water (inches)
0	27
5	22
7	20
12	15

1. Graph the data. Connect the data values with a line. Be sure to label your axes.



2. **Assuming that the water evaporated at the same rate every day, use your graph to determine the water level the day he was supposed to start data collection.**

3. **Assuming that the water evaporated at the same rate every day, use your graph to determine when the water level was**

   a. **30 inches.**

   b. **12 inches.**

   c. **5 inches.**

   d. **0 inches.**

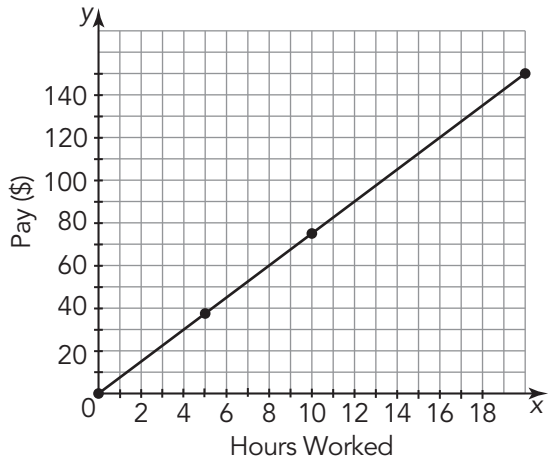
4. **Explain why you should or should not extend your graph into Quadrant IV.**

NOTES

ACTIVITY
3.6

Hard Working in the Hardware Store

Your friend Aidan got a job working at the local hardware store. He created the graph shown to track how much money he makes for a given number of hours.



1. Create a table of values for Aidan's graph.

Hours Worked	Pay ($)
0	
5	
10	
15	
20	

2. How can you tell, by looking at the graph, whether the graph displays equivalent ratios? If it does, what is the ratio, or rate, displayed in the graph?

3. Define variables for the hours worked and Aidan's pay.

4. Write an equation to describe Aidan's graph.

5. Use the tool of your choice—equation, graph, or a table—to answer each equation.

   a. Approximately how much money did Aidan make if he worked 15 hours this week?

   b. Determine the exact amount of money Aidan made if he worked 12 hours this week.

   c. Approximately how many hours did Aidan work if he made $50 this week?

   d. Determine the exact number of hours Aidan worked if he made $152.50 this week.

   e. How did you decide which tool to use to answer each question?

ACTIVITY
3.7

Broken Yardstick

Jason and Liliana need to measure some pictures so they can buy picture frames. They looked for something to use to measure the pictures, but could find only a broken yardstick. The yardstick was missing the first $2\frac{1}{2}$ inches.

They both thought about how to use this yardstick.

Lilianna said that all they had to do was measure the pictures and then subtract $2\frac{1}{2}$ inches from each measurement.

1. Is Lilianna correct? Explain your reasoning.

2. They measured the first picture's length and width to be 11 inches and $9\frac{1}{2}$ inches. What are the actual length and width?

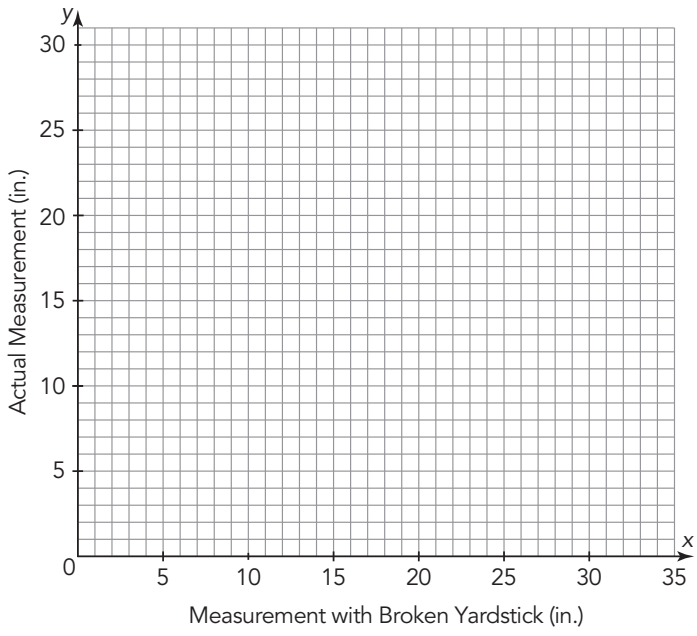
3. Define variables for a measurement with the broken yardstick and the actual measurement.

4. Write an equation that models the relationship between the variables.

5. **Complete the table of values for the measurement on the yardstick and the actual measurement.**

Measurement with Broken Yardstick (in.)	Actual Measurement (in.)
11	
$9\frac{1}{2}$	
$25\frac{3}{4}$	
21	
$18\frac{5}{8}$	
	12
	$29\frac{1}{8}$
	$6\frac{7}{8}$

6. **Use the table to complete the graph of the actual measurements versus the measurement taken with the broken yardstick.**



7. **Would it make sense to connect the points on this graph? Explain why or why not.**

8. Suppose the yardstick was broken at 5 inches instead of $2\frac{1}{2}$ inches.

   a. Write the new equation for the relationship between the actual measurement and the measurement from the broken yardstick.

   b. Sketch a graph of the actual measurements versus the measurement taken with the new broken yardstick on the graph with the original yardstick.

   c. What do you notice about the two graphs?

   d. What is the meaning of the *x*-intercept—the point with a *y*-coordinate of zero—on each graph?

ACTIVITY
3.8 The Diver

A freediver is a person who dives into the ocean without the use of any breathing device like scuba equipment. William Trubridge holds the record for freediving. In 2016, he broke his own record and dove almost 407 feet into the ocean! Suppose you plan to train as a freediver and want to beat Trubridge's record.

1. **What are some questions you would ask of Trubridge about his dive?**

2. **Assume that Trubridge ascended and descended at the same rate of 2.97 feet per second to help you determine how much time you need to be able to hold your breath to beat Trubridge's record.**

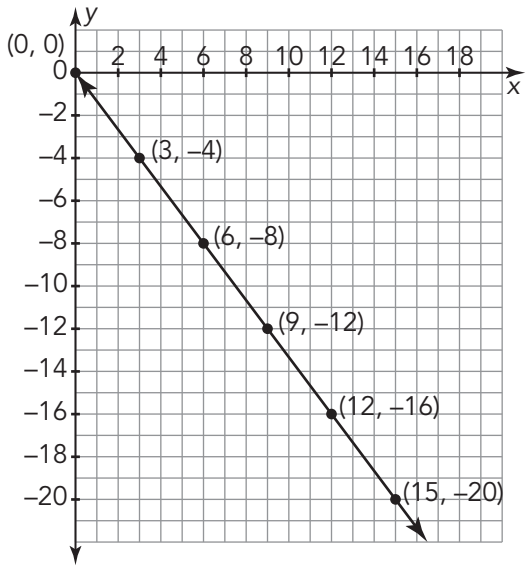
TALK the TALK

Your Turn!

You and your group should prepare a presentation for this problem.

1. Create a situation that can be modeled by the graph.

Write at least 3 sentences for what you want to say during your presentation.



- Be sure to determine the ratio, or rate, for how the variables change in relation to each other.
- Describe the meaning of each point on the graph.
- Define variables for the independent and dependent quantities based on your situation.
- Write an equation to represent the problem situation.

Assignment

Write

Give an example of when you might want to use an equation to answer a question and another example of when you might want to use a graph.

Remember

Graphs, tables, equations, and scenarios provide different information and allow for various levels of accuracy when solving problems.

Practice

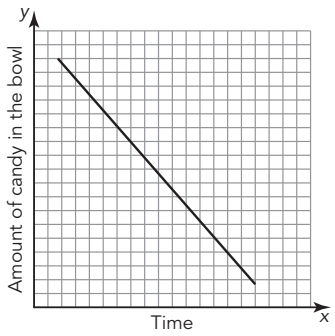
1. The gravitational pull of the Moon is not as great as that of Earth. In fact, if a person checks his weight on the Moon, it will be only $\frac{1}{6}$ of his weight on Earth.
   a. If a person weighs 186 pounds on Earth, how much will he weigh on the Moon? How many pounds different from his actual weight is that?
   b. Complete the table of values for a person's weight on Earth, weight on the Moon, and difference of the two weights. Use negative numbers when the weight is less than the person's earth weight.

Weight on Earth (lb)	186	168		198		
Weight on Moon (lb)			29		21	24
Weight Differential	−155					

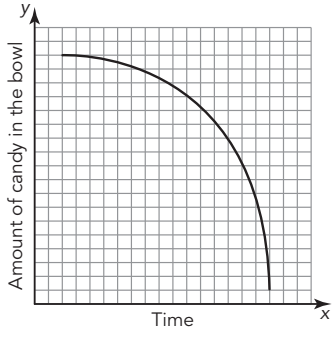
   c. Graph the weight differential versus the weight on Earth. Be sure to label your axes.

2. To keep her students relaxed and focused during tests, Ms. Chappell puts small bowls of candy on each of their desks. Write a short story to describe each graph.

a.



b.



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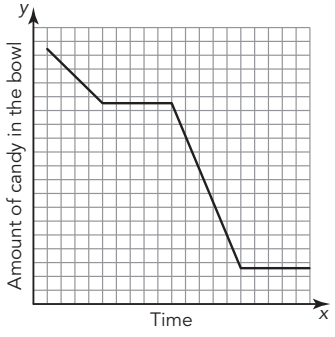
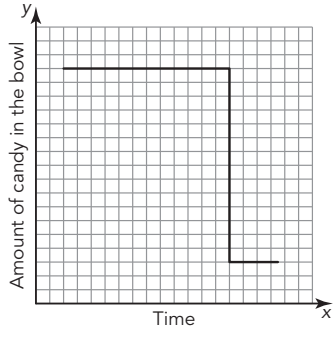
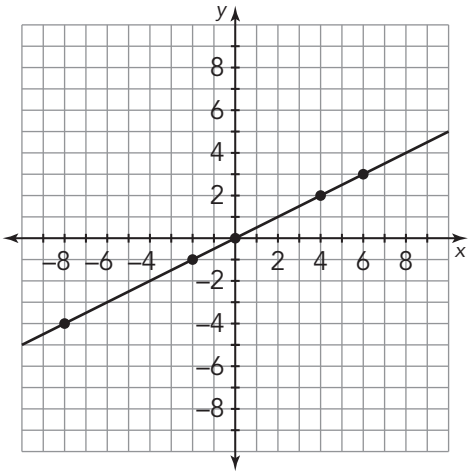


d.



3. The following graph shows the average temperature, in degrees Celsius, in Fairbanks, Alaska. The *x*-axis represents time in days from January 1, and the *y*-axis represents degrees Celsius.



a. Label the axes with the independent and dependent quantities and their units.

b. Create a table of values for the points on the graph and describe the meaning of each.

x	*y*	Meaning

c. At what rate did the temperature increase?

d. Define variables for the quantities that are changing, and write an equation for this situation.

4. Sarina's dog, Bruno, has to go on a diet! Sarina puts Bruno on a diet plan of daily exercise and a special type of dog food. She estimates Bruno will lose $1\frac{1}{2}$ pounds per week on this plan.

a. How many pounds does Sarina estimate Bruno will lose in 2 weeks? In $8\frac{1}{2}$ weeks?

b. Define variables for the independent and dependent quantities for this situation.

c. Write an equation for this situation. (Because Bruno is losing weight, the number of pounds he loses will be defined as a negative value.)

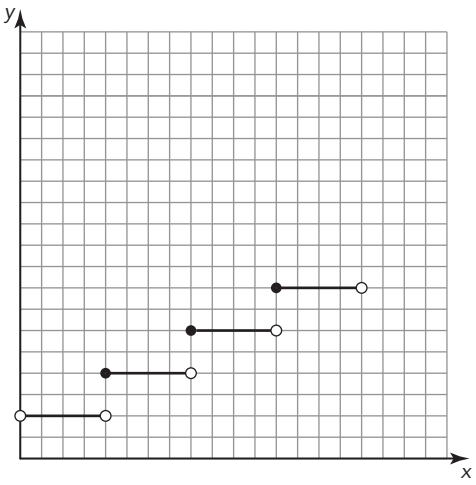
d. Create a table of values for the situation.

e. Complete a graph of the situation.

f. Explain what points in Quadrant I would mean for Bruno.

Stretch

Tell a story to describe the graph.



Review

1. The vertices of a polygon are given. Plot the points on a coordinate plane and connect the points in the order they are listed. Then determine the area of the polygon.

   (−4, −1), (−3, −2), (10, −2), (3, 0), (0, 4), (−2, 3)

2. Create a scenario to fit each numeric expression.

   a. $|-3 + 21|$
   b. $|8 - 3|$

3. Evaluate each expression for the given values.

   a. $5.2r + 1.2$, when $r = 1.5$ and 4.1
   b. $\frac{1}{2}t + \frac{3}{4}$, when $t = \frac{2}{3}$ and $\frac{9}{5}$