

There Are Many Paths ...

Problem Solving on the Coordinate Plane

3

MATERIALS

None

Lesson Overview

Students apply their knowledge of plotting rational numbers on the coordinate plane, interpreting points on the coordinate plane, creating tables of values, and writing and solving equations to solve a variety of problems situated on the coordinate plane. Students model real-life situations and analyze data. They select which representation to use for specific problems.

Grade 6 The Number System

Apply and extend previous understandings of numbers to the system of rational numbers.

8. Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Grade 6 Expressions and Equations

Represent and analyze quantitative relationships between dependent and independent variables.

9. Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation.

Essential Ideas

- Multiple representations such as situations written in words, equations, tables, and graphs can be used to solve problems.
- Graphs other than lines can be used to model real-life situations.
- Graphs can be used to interpret data and changes in data.
- There are advantages and disadvantages to using different mathematical tools to solve problems.

Lesson Structure and Pacing: 4 Days

Day 1

Engage

Getting Started: Emma's Birthday

Students analyze a graph that crosses three quadrants. They create a scenario from the graph, explain the meaning of values on the graph, and write an equation for the graph.

Develop

Activity 3.1: Weigh In

Students plot data in Quadrant I, transform the data, and plot the transformed data in Quadrants I and IV. They review independent and dependent quantities, make decisions about which representations to use to answer analysis questions, and interpret data. Students have opportunities to review using absolute value equations to determine the distance between two rational numbers.

Activity 3.2: An Interesting Day in South Dakota

Students analyze a situation involving temperature swings from positive to negative temperatures repeatedly during a day. They determine the number of quadrants needed to plot the data and answer questions about the data.

Day 2

Activity 3.3: No Place Like Home

Students are given a variety of graphs to interpret. First, students analyze a broken line (piecewise) graph that describes a day in the life of a student. They then create a situation for a graph with no numbers. Next, given a scenario, students create a graph. Finally, students analyze four graphs that illustrate a different relationship between the time and the volume of air contained while a balloon is being blown up. Students will match a verbal scenario with the appropriate representative graph.

Activity 3.4: Pool Level

Students are given a graph containing several points and use the graph to create a table of values and interpret the situational meaningfulness of specific points on the graph. They then define variables and write an equation to represent the situation.

Day 3

Activity 3.5: Water in the Bucket

Students use a scenario and a table of values to construct a graph of the rate of water evaporating from a bucket. They use the graph to determine specific values for each quantity. Students interpret the meaning of points in different quadrants of the graph.

Activity 3.6: Hard Working in the Hardware Store

Beginning with a graph of hours worked and pay, students create a table of values and an equation for the situation. They use the different representations to answer questions about the problem.

Activity 3.7: Broken Yardstick

A yardstick has 2.5 inches broken off of the end and it is used to measure distances. Students determine the actual measurements by subtracting 2.5 inches from the measurements they determined using the broken yardstick and write an equation to represent the situation. Students use the equation to complete a table and the table of values is used to graph the situation.

Day 4**Activity 3.8: The Diver**

Students are provided with a situation and pose questions about the situation. Then they devise and carry out a strategy, using different representations, to solve the problem.

Demonstrate**Talk the Talk: Your Turn!**

Students work in pairs or groups to analyze a graph, determine a scenario for the graph, and determine an equation. Then they present their scenarios to the class.

Facilitation Notes

In this activity, students analyze a linear graph drawn in three quadrants within the context of a problem situation.

At the end of Module 3, students used equations and first quadrant graphs to model quantitative relationships. This activity is intended to help students recall what they learned and build on that knowledge.

This is students' introduction to graphs of lines that include points with negative coordinates. Provide students with an opportunity to make conjectures and critique each other's reasoning about the graph. Students may struggle to develop the equation.

Ask the students to read through all four questions and then think about the graph. Provide students with time to process the questions and think about the graph. Have students turn to a partner and share their ideas. Have students share their interpretations, explanations, and equations as a class.

Questions to ask

- What is the independent variable?
- What is the dependent variable?
- What is the x -intercept?
- What is the y -intercept?
- What does the point $(-5, 10)$ mean in this context?
- What does the point $(0, 5)$ mean in this context?
- What does the point $(1.5, 3.5)$ mean in this context?
- What does the point $(5, 0)$ mean in this context?
- What does the point $(10, -5)$ mean in this context?
- As the independent variable increases in value by 1 unit, how does the dependent variable change?

Summary

Four quadrants can be used to model real-life situations that include positive and negative numbers.

Activity 3.1

Weigh In



DEVELOP

Facilitation Notes

In this activity, students plot data in Quadrant I, transform the data, and plot the transformed data in Quadrants I and IV. They review independent and dependent quantities, make decisions about which representations to use to answer analysis questions, and interpret data. Students have opportunities to review using absolute value equations to determine the distance between two rational numbers.

Have students complete Questions 1 and 2 and then discuss as a class.

Differentiation strategies

- If students work in groups, assign half of the group to complete Question 1 and half of the group to complete Question 2. Groups can share and compare, answering Questions 3 and 4 together. It is important, however, for all students to have at least a sketch of both graphs.
- Assign half of the class Question 1 and the other half of the class Question 2. Everyone completing Question 1 (or Question 2) should ensure that they agree on their graphs. Then pair a student with Question 1 completed with a student that has Question 2 completed to share and compare, and then answer Questions 3 and 4.

Of Questions 1 and 2, Question 2 is a little more difficult. In Question 2, part (a), students should subtract the smaller number from the larger and apply the negative sign on the appropriate differentials. Students are not expected to use integer operation rules.

Differentiation strategies

For students who struggle with interpreting the graph,

- They may see that all their y-values are close together, and it is difficult to tell the difference between them. Explain how to include a scale break that indicates that the values displayed do not include all of the values that exist on the number line, i.e., pounds.
- Provide a scaled and labeled graph to allow students to focus on the new concepts of this lesson.

Questions to ask

- What quantities are being compared?
- Which quantity depends on the other?
- How did you decide which quadrants you will need in order to graph the data?
- How did the signs of the numbers in the table help you decide which quadrants are needed in order to graph the data?
- How did you determine the weight differentials in Question 2?

After students have graphed the data in Questions 1 and 2, they should compare the graphs and the data.

Misconception

Some students may decide to connect the points as they move horizontally in order to visualize the weight change, creating a line graph. If that occurs, students may be confused when comparing this activity to Module 3, Topic 4 Activity 4.1 with Deazia’s average swimming speed; in this activity the coach drew a line to determine average speed rather than connect the points. Explain that the question asked (determining average rate or calculating change) determines whether to connect the points or not.

Have students complete Questions 3 and 4 and then discuss as a class. In Question 4, focus on the advantages and disadvantages of the representations. Some students may use the graph to get an idea for the differences and then turn to the table for the exact numbers. Others may rely on only one representation. Allow students to explain why they chose their chosen strategy.

Questions to ask

- Describe the pattern on the points in each graph. What do you notice?
- Which axis is the x-axis? What is the label on the x-axis in each graph?
- Do you think you could write equations for these graphs? Explain.
- Which type of graph could be generalized for any player, regardless of weight, to show the coach how well the player is sticking to his target weight?
- How can you use absolute value equations with the weight differentials to determine the weight changes?
- What is an advantage of using the weight differentials over the actual weights when calculating the weight change?
- How can the graphs help you see the greatest weight changes?
- How can the tables help you see the greatest weight changes?
- Do you think the major swings in Julio’s weight is healthy?

Summary

Positive and negative numbers can be used to model and analyze real-life situations.

Activity 3.2

An Interesting Day in South Dakota



Facilitation Notes

In this activity, students analyze a situation involving a table of values describing temperature swings which include positive and negative temperatures. They plot the data and answer questions about the situation.

Have students work with a partner or in groups to complete Questions 1 through 5, then share responses as a class. Encourage students to use absolute value equations to calculate temperature changes. For example, 12:00 P.M. to 12:05 P.M.: $|15.6| + |-10.6| = 26.2$ degree temperature change.

Differentiation strategies

- For students who struggle with scaling the graph, discuss different ways to represent time on the x-axis.
- Provide a scaled and labeled graph.

Questions to ask

- What quantities are being compared?
- Which quantity depends on the other?
- How did you decide which quadrants you will need in order to graph the data?
- How did the signs of the numbers in the table help you decide which quadrants are needed in order to graph the data?
- How is this graph similar to or different from the graphs in Julio's Weigh In?
- Describe the pattern on the points on the graph. What do you notice?
- How can you use absolute value equations to calculate the temperature swings?
- How can the graph help you see the greatest temperature swing?
- How can the table help you see the greatest temperature swing?
- Do you think you could write an equation for this graph? Explain.

Summary

Positive and negative numbers can be used to model and analyze real-life situations.

Activity 3.3

No Place Like Home



Facilitation Notes

In this activity, students are given a variety of graphs to interpret including a piecewise graph. They create a situation for a graph with no numbers, and given a scenario, they create a graph. Also, students analyze four graphs that illustrate a different relationship between the time and the volume and match a verbal scenario with the appropriate representative graph.

Have a student read the introduction aloud. Ask students what they think is happening in the graph. Provide students with an opportunity to consider the graph on their own before completing Questions 1 and 2 as a class.

Questions to ask

- What is the independent quantity in this situation?
- What do the negative numbers on the x-axis represent with respect to the situation?
- What do the negative numbers on the y-axis represent with respect to the situation?
- What does it mean if time is represented by a negative number?
- What does it mean if the distance from home is represented by a negative number?
- Over time, is the distance from home increasing or decreasing? How do you know?
- What aspect of the graph tells you that the distance from home over time is increasing?
- What does the point $(0, 0)$ mean in the situation?
- According to the graph, were you home at 12:00 P.M.?
- What time corresponds to $x = -6$? What would you expect to happen *prior* to that time?
- What time corresponds to $x = 10$? What would you expect to happen *after* that time?

Have students complete Questions 3 with a partner or in groups and share responses as a class.

Differentiation strategy

For students who struggle, suggest time and distance for the independent and dependent variables, respectively.

Questions to ask

- What independent and dependent quantities did you choose?
- What units did you use for each quantity?
- What is your scale for each axis of your graph?
- What is the meaning of $(0, 0)$ in your scenario?
- What is the meaning of the x-axis?
- What is the meaning of the y-axis?
- There are four distinct points on the graph. What is the meaning of each of those points in your scenario?

Have students complete Questions 4 with a partner or in groups and share responses as a class.

Questions to ask

- What are the independent and dependent quantities?
- What are the units for each quantity?
- What scale did you use for each axis?
- What is the meaning of $(0, 0)$ on the graph?
- What is the meaning of the x-axis?

- What is the meaning of the y-axis?
- How many distinct points are described in the scenario? What is the meaning of each one?

In Question 5, students encounter graphs that include curves, which may be new for students. Ask students what they think is different about a scenario graphed using lines and a scenario graphed with curves. Then have students complete Questions 5 and 6 with their partner or in groups and share responses as a class.

Questions to ask

- What does the highest point on Graph 1 represent?
- What aspect of Graph 1 tells you the balloon did not pop?
- What is the volume of air in the balloon in Graph 2 at 8 seconds?
- When is the balloon in Graph 3 halfway filled?
- What does the highest point on Graph 4 represent?
- What is the difference between Graph 2 and Graph 3?
- What aspect of the graph tells you that the balloon popped?
- What aspect of the graphs tells you that the balloon was tied off?
- What aspect of the graph tells you the air was let out of the balloon?

Differentiation strategy

To extend the activity, provide balloons and have students model what is going on in each graph.

Summary

Four-quadrant graphs can be used to describe a variety of scenarios, both with and without numbers.

Activity 3.4

Pool Level



Facilitation Notes

In this activity, students are given a graph containing several points and use the graph to create a table of values and interpret the situational meaningfulness of specific points on the graph. They also define variables and write an equation to represent the situation.

After completing Activity 3.3, students should be able to complete this activity individually. Have students complete Questions 1 through 7. Share responses as a class.

Questions to ask

- What is the independent quantity in this situation?
- What do the negative numbers on the x-axis represent with respect to the situation?
- What do the negative numbers on the y-axis represent with respect to the situation?
- What does it mean if time is represented by a negative number?
- What does it mean if the water level of the pool is represented by a negative number?
- Over time, is the water in the pool increasing or decreasing? How do you know?
- What aspect of the graph tells you that the water level in the pool is increasing?
- What does the point $(0, 0)$ mean in the situation?
- Does the water level increase at a constant rate? How do you know?
- What aspect of the graph tells you that the water level increases at a constant rate?
- What would it mean with regards to the water level if the graph continued infinitely?
- Which representation did you use to determine when the pool was 3 inches above the desired fill level?
- How could you use a different tool to determine when the pool was 3 inches above the desired fill level?

Summary

Tables, graphs, and equations that contain positive and negative numbers are representations that can be used to answer questions about real-life situations.

Activity 3.5

Water in the Bucket



Facilitation Notes

In this activity, students use a scenario and a table of values to construct a graph of the situation. They use the graph to determine specific values for each quantity, and interpret the meaning of points in different quadrants of the graph.

Have students work with a partner or in groups to complete Questions 1 through 4. Share responses as a class.

Questions to ask

- What varies in this situation?

- Which variable depends on the other variable?
- What does the x -value of each point represent in the problem situation?
- What does the y -value of each point represent in the problem situation?
- Could another point be located between each of the two points on the graph? Explain.
- As the *Days Since Sunday* increase, how does the *Height of the Water* change?
- How many days *before* Sunday was Damon supposed to begin his data collection? What x -value would this be?
- How did you use the graph to determine when the water level would be 30 inches? 12 inches? 5 inches? 0 inches?
- What is the meaning of the x -intercept in this problem?
- What is the meaning of the y -intercept in this problem?
- What would points plotted in Quadrant II mean in regards to the context?
- What would points plotted in Quadrant III mean in regards to the context?
- What would points plotted in Quadrant IV mean in regards to the context?

Summary

Although scenarios with positive and negative numbers can be modeled with four quadrant graphs, points in all of the quadrants do not always make sense in the context of the problem.

Activity 3.6

Hard Working in the Hardware Store



Facilitation Notes

In this activity, students use a graph to create a table of values and an equation for the situation. Students use the different representations to answer questions about the problem.

The goal of this activity is for students to see when they might want to use different representations to solve problems. Have students work with a partner or in groups to complete Questions 1 through 5. Share responses as a class.

Questions to ask

- What are the independent and dependent quantities in this situation?
- If you know the number of hours worked, how do you determine the amount earned?

- If you know the amount earned, how do you determine the number of hours worked?
- Which variable represents the amount earned?
- Which variable represents the number of hours worked?
- Is it easier to determine the amount earned or the number of hours worked? Why?
- What does the x -value of each point represent in the problem situation?
- What does the y -value of each point represent in the problem situation?
- What aspect of the graph tells you that the overall pay is increasing?
- How can you tell if a graph displays equivalent ratios?
- What are the characteristics of graphs that display equivalent ratios?
- At what rate is Aidan being paid? How can you determine that from the graph?
- What do you notice about the location of the points on the graph? Do you see a pattern?
- Could another point be located between each of the two points on the graph? Explain.
- Which tools, or representations, are most appropriate if a question asks for an *exact* amount?
- Which tools, or representations, can be used if a question asks for an *approximate* amount?

Summary

Any representation is helpful when approximating solutions, however solving equations is often most appropriate when an exact solution is required.

Activity 3.7

Broken Yardstick



Facilitation Notes

In this activity, a yardstick has 2.5 inches broken off of the end and it is used to measure distances. Students write an equation to represent the situation, use the equation to complete a table of values, and use the table of values to graph the situation.

Consider bringing in a broken yardstick to aid the discussion.

Have a student read the introduction aloud. Ask students to work with a partner or in groups to complete Questions 1 and 2. Share responses as a class.

Questions to ask

- If 2.5 inches of the yardstick is broken off, why wouldn't you add 2.5 inches to your answer to get the actual measurement?

- If you know the measurement taken with the broken yardstick, how do you determine the actual measurement?

Ask students to work with a partner or in groups to complete Questions 3 through 5. Share responses as a class.

Questions to ask

- What varies in this situation?
- What measurement remains the same in this situation?
- Which variable depends on the other variable?
- Which variable represents the actual measurement?
- Which variable represents the measurement taken with the broken yardstick?
- How is the equation for this situation different from the equations you have written for other problem situations?
- If you know the measurement taken with the broken yardstick, how do you determine the actual measurement?
- If you know the actual measurement, how do you determine the measurement taken with the broken yardstick?
- Is it easier to determine the actual measurement or the measurement taken with the broken yardstick? Why?
- How do you add or subtract mixed numbers with different denominators?

Have students work with a partner or in groups to complete Questions 6 and 7. Have students check their graphs with another set of students' graphs. Look for student graphs that are nonlinear due to inaccurate graphing.

Questions to ask

- What does the x-value of each point represent in the problem situation?
- What does the y-value of each point represent in the problem situation?
- What do you notice about the location of the points on the graph? Do you see a pattern?
- Could another point be located between each of the two points on the graph? Explain.

Have students complete Question 8 with a partner or in groups. Share responses as a class.

Questions to ask

- Where does $2\frac{1}{2}$ appear in the work or in the answers?
- How might the equation be different if the number of inches broken from the yardstick was different?
- How might the graph be different if the number of inches broken from the yardstick was different?
- What does the x-value of each point represent in the problem situation?
- What does the y-value of each point represent in the problem situation?

Summary

You can solve problems with rational numbers on the coordinate plane.

Activity 3.8

The Diver



Facilitation Notes

In this activity, students are provided with a situation and pose questions about the situation. They then devise and carry out a strategy, using different representations, to solve the problem.

When engaging in mathematical modeling, you sometimes need to make assumptions for unknowns or, as new ideas are being learned, to simplify the scenario so that students can model them. This problem is left open to allow students to make a plan, decide on a course of action, carry out their plan, revise the plan, and write a conclusion.

Questions to Ask

- How deep will he be after 10 seconds? 25 seconds? 1 minute? $1\frac{1}{2}$ minutes?
- How did you determine how deep the diver will be in 10 seconds?
- What are the independent and dependent quantities in this situation? Define the variables and state their units of measurement.
- How did you decide which quantity was the independent quantity?
- Write an equation for Trubridge's descent in this problem situation.
- How is this equation different from other equations you have written?
- Create a table of values for this situation.
- In the table of values, as the independent variable increases, what is happening to the dependent variable?
- How many and which quadrants are needed to accurately graph this situation?
- Graph the equation or the table.
- Describe the graph of the equation.
- How is the graph of the equation different from most of the other equations you have previously graphed?
- What is the uppermost point on this graph? Considering the situation, can other points be located above this point? If so, what are they? What do they mean?
- Considering the situation, will the graph of this line continue to negative infinity? Explain.
- At what depth was William Trubridge at 2 minutes into his world-record

breaking dive?

- Use your graph to determine his depth.
- Use your equation to determine his depth.
- Compare your answers from your graph and from your equation. Which answer is more accurate? Why?
- Explain why the points should be connected in this graph. For what values of time should the graph begin and end?

Search online for more information on William Trubridge.

Summary

Positive and negative numbers can be used to model and answer questions about real-life situations.

Talk the Talk: Your Turn!

DEMONSTRATE

Facilitation Notes

In this activity, students analyze a graph, determine a scenario for the graph, and write an equation. Have students work in pairs to complete this activity. Require students to script out their presentation. Each student should write out their presentation in the space provided. Share responses as a class.

Questions to ask

- Is the graph increasing or decreasing?
- What is the rate of increase or decrease?
- Is the relationship between consecutive y -values additive or multiplicative?
- Will the coefficient be positive or negative?
- Describe how the y -values change as the x -values increase.
- Describe how the y -values change as the x -values increase by 1 unit.
- What quantities and units will you relate in your situation?

As students work, look for

- Confusion over the independent and dependent variables.
- Failure to recognize that the y -values decrease by 4 for each 3 unit increase in the x -values.

Differentiation strategies

- Provide students with suggested independent and dependent quantities.
- Provide students with a suggested meaning for 1 data point.

Summary

One graph can be used to describe a variety of scenarios.

There Are Many Paths ...

3

Problem Solving on the
Coordinate Plane

WARM UP

Solve each equation.

1. $120 + h = 315$

2. $w - 17 = 38$

3. $\frac{c}{5} = 12$

4. $169 = 13w$

LEARNING GOALS

- Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane.
- Interpret the meaning of points plotted on the coordinate plane.
- Use equations to solve real-world problems.
- Use graphs relating an independent and dependent quantity changing in relationship to one another to solve real-world problems.
- List advantages and disadvantages of different representations for solving real-world and mathematical problems on the coordinate plane.

Now that you understand how to plot points in all four quadrants of the coordinate plane, you can solve many more types of problems than you could previously. How can you use graphs and equations to solve problems?

LESSON 3: There Are Many Paths ... • M4-87

Warm Up Answers

1. $h = 195$

2. $w = 55$

3. $c = 60$

4. $13 = w$

Answers

1. The graph shows the number of days since Emma's birthday and the number of birthday cards she received on specific days.
2. For example, the point $(0, 5)$ means that on Emma's birthday, she received 5 cards. The point $(5, 0)$ means that 5 days after her birthday, she received 0 cards. And the point $(-5, 10)$ means that 5 days before her birthday, she received 10 cards.
3. All of the values do not make sense. Days are measured only in integers, so x-coordinates with fractional values do not make sense. Also, Emma can't receive a negative number of birthday cards, so the values in Quadrant IV do not make sense. Because Emma probably did not start receiving birthday cards more than a week before her birthday, any x-coordinates less than -7 are unlikely to fit this situation.
4. The equation is $y = -x + 5$, where x = number of days since Emma's birthday and y = number of birthday cards received.

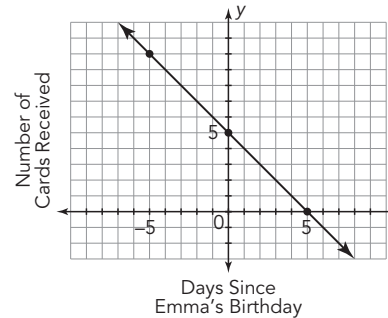
ELL Tip

Several exercises in this lesson require students to infer the concept of continuity. Some English Language Learners may have difficulty grasping

Getting Started

Emma's Birthday

Analyze the graph.



1. Explain what you can determine about the situation from the graph.
2. What do the plotted points mean in terms of this situation?
3. Do all of the values on the line make sense in terms of the situation?
4. Can you determine an equation for the graph?

M4-88 • TOPIC 2: The Four Quadrants

and articulating how a graph shows the continuity of the real-world situation it models. Have advanced English Language Learners engage in a Reciprocal Teaching exercise by explaining their answers to beginners. Beginners repeat back what they heard, and the advanced English Language Learner makes corrections to refine understanding.

ACTIVITY
3.1

Weigh In



Julio is a wrestler for his high school team. Although he does not wrestle during the 12 weeks of summer, his coach would like him to stay around 140 pounds so that he doesn't have to work so hard during the season to stay in his 142-pound weight class. Julio charted his weight over the summer.

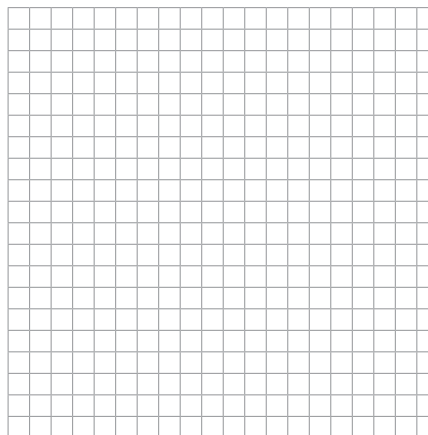
Week	1	2	3	4	5	6	7	8	9	10	11	12
Weight	144.5	142.1	138.5	139.5	137.5	141.5	136.25	137.2	140	141.3	138.5	135
Weight Differential												

1. Consider the table shown.

a. Which quantity is the independent quantity and which is the dependent? Explain your reasoning.

b. What is the unit for each quantity?

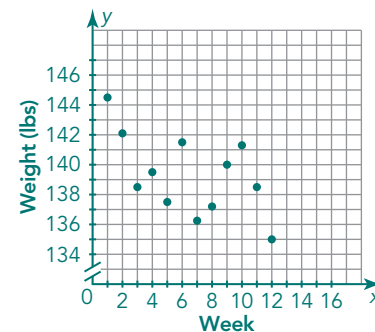
c. Which quadrant(s) will you need in order to plot Julio's data? Draw and label your axes. Then graph the data.



LESSON 3: There Are Many Paths ... • M4-89

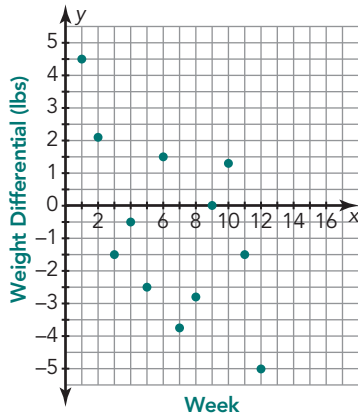
Answers

- 1a. The week is the independent quantity and the weight is the dependent quantity. The weight depends on the week.
- 1b. The units for weeks are the number of weeks. The units for weight are pounds.
- 1c. Quadrant 1

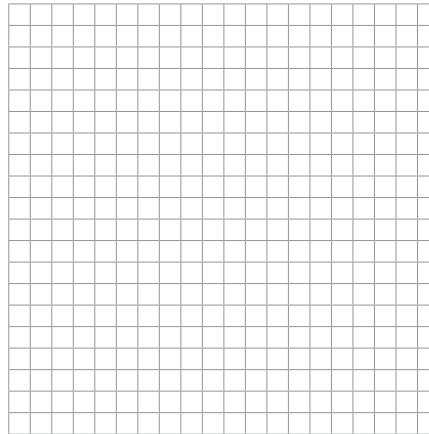


Answers

- 2a. See table below.
- 2b. The dependent quantity is the weight differential, or weight difference from 140 pounds.
- 2c. Quadrants I and IV are needed.



2. The coach was impressed with Julio's data collection, but he was interested in how much Julio's weight varied from 140 pounds each week.
- Complete the last row of weight differentials, the differences of Julio's weight from 140 pounds. Use negative numbers when the weight is below 140 pounds and positive numbers when his weight is above 140 pounds.
 - What is the dependent quantity in this situation?
 - Which quadrant(s) will you need in order to plot Julio's data for the coach's request? Draw and label your axes, including the units. Then graph the data.



M4-90 • TOPIC 2: The Four Quadrants

Week	1	2	3	4	5	6	7	8	9	10	11	12
Weight	144.5	142.1	138.5	139.5	137.5	141.5	136.25	137.2	140	141.3	138.5	135
Weight Differential	4.5	2.1	-1.5	-0.5	-2.5	1.5	-3.75	-2.8	0	1.3	-1.5	-5

3. Compare the two approaches taken by Julio and his coach.
- Compare the independent and dependent quantities.
 - Compare the graphs. What do you notice about the patterns of the points?
 - Explain the meaning of the x -axis in each approach.
 - Why do you think the coach preferred his approach over Julio's approach?
4. Use the table and graphs to answer each question.
- Between which two consecutive weeks did Julio's weight change the most? What was the weight change?
 - What is the difference between Julio's highest weight and his lowest weight?
 - Which representation—table, Julio's graph, the coach's graph—did you use to answer the questions? Why did you make those choices?
 - If you were Julio's coach, what advice would you give Julio?

LESSON 3: There Are Many Paths ... • M4-91

Answers

- 3a. The independent quantities are both the number of weeks. Both dependent quantities are in terms of pounds; Julio's is actual weight and the coach's is the number of pounds from 140 pounds.
- 3b. The pattern of the points is the same in the two graphs.
- 3c. Julio's approach: The x -axis is a weight of 0 pounds. Coach's approach: The x -axis is the target weight of 140 pounds.
- 3d. The coach's graph shows more clearly how close Julio is staying to his target weight. This same kind of graph could be used for all the wrestlers, regardless of their target weight.
- 4a. Weeks 6 and 7. $|1.5| + |-3.75| = 5.25$ pounds
- 4b. Weeks 1 and Weeks 12: $|4.5| + |-5| = 9.5$ pounds
- 4c. Answers will vary. I used the table because I could read the differentials and use absolute value equations to efficiently answer the questions.
- 4d. I would advise Julio to stabilize his diet and exercise. He should not have weight swings like this in a summer, especially week to week.

Answers

1. Quadrants I and IV are needed. All of the times are positive but the temperatures are positive and negative. We need to be able to plot $(+, +)$ and $(+, -)$ points.
2. See graph on next page.

ACTIVITY 3.2

An Interesting Day in South Dakota



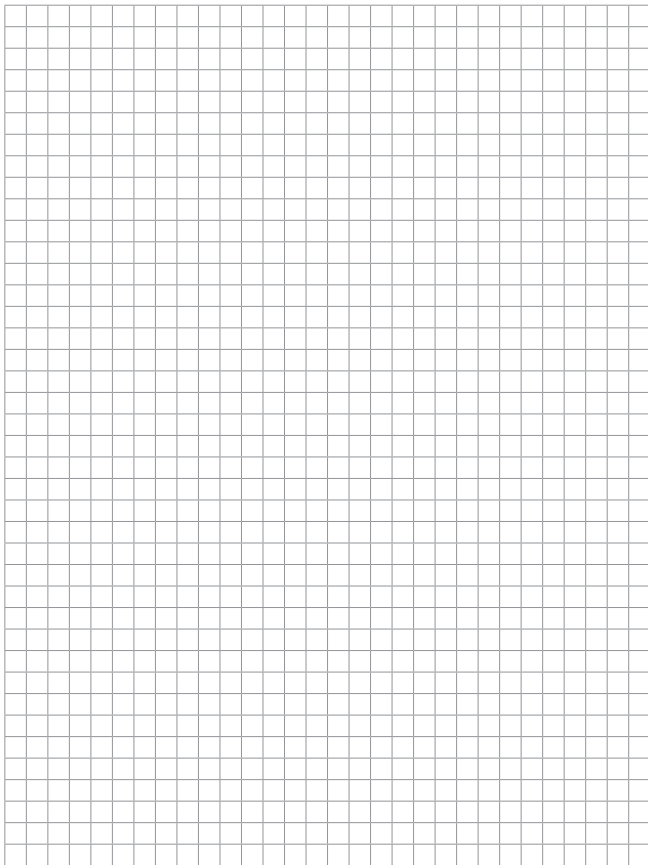
An interesting day of temperature changes occurred in Rapid City, South Dakota, on January 22, 1943. The table shows the temperature changes that happened throughout the day.

Time	Temperature (°C)
10:30 A.M.	-6.7
10:35 A.M.	13.3
12:00 P.M.	15.6
12:05 P.M.	-10.6
12:35 P.M.	-9.4
12:40 P.M.	10
2:20 P.M.	14.4
2:25 P.M.	-8.3

Create a graph of the temperature changes.

1. Which quadrants do you need for your graph? Explain your reasoning.

2. Draw and label the axes for the graph. Then graph the data and connect consecutive points.



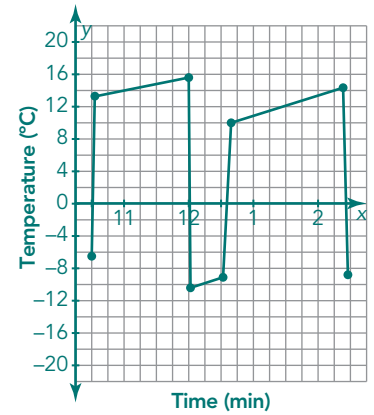
NOTES

3. Between which two times was the temperature swing the greatest?

4. Describe the pattern. Why is this called an "interesting" day?

Answers

2.



3. The temperature swing was the greatest in the 5 minute intervals that look almost like vertical lines. The longest near-vertical line is between 12:00 PM and 12:05 PM.

4. The temperature was mostly in the teens but there were short periods of time when the temperature would drop drastically. From the graph, it appears that the drops in temperature did not last very long.

Answers

1.

Meaning
Two hours ago, at 10:00 AM, I was 4 blocks south of my house.
At 12:00 PM, I was at home.
At 4:00 PM, I was 2 blocks north of my house.
At 8:00 PM, I was 4 blocks north of my house.

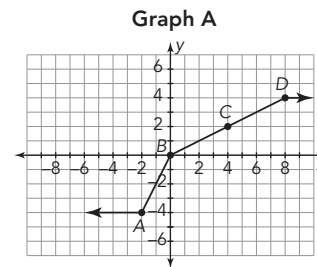
2. Sierra is correct. You do not have enough information to know exactly what happened after 10:00 PM or before 6:00 AM, but it is likely that you would have started at home and ended your day at home, which is represented as the x -axis.

ACTIVITY 3.3

No Place Like Home



Suppose this graph summarizes your day. The x -axis of this graph represents time in minutes from 12:00 P.M., and the y -axis represents your distance from home in blocks. Locations north of your house are positive, and locations south of your house are negative. A point at the origin represents you being home at 12:00 P.M.



1. Describe the meaning of each of the four labeled points.

x	y	Meaning
-2	-4	
0	0	
4	2	
8	4	



2. Adrian and Sierra are discussing how the graph should look before $x = -6$ and after $x = 10$. Adrian thinks he should draw arrows to indicate that the graph continues to the left and right, respectively. Sierra disagrees and thinks they should draw segments back to the x -axis. Who is correct?

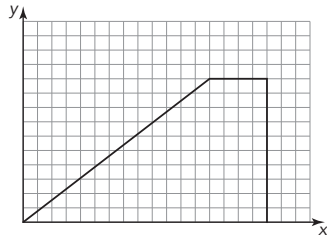
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ELL Tips

Help English Language Learners build stories by listing observations about the graphs. Ask the following questions:

- How are the x -axes labeled on each graph?
- How are the y -axes labeled on each graph?

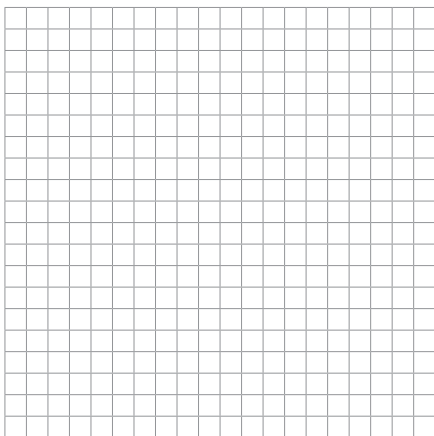
Let's consider another graph.



3. Write a possible scenario for this graph. Be sure to specify the units and the meaning of the origin for your scenario.

Natasha and her family took a 3-day trip to her grandmother's house. On the first day, they drove 300 miles. On the second day, they drove 350 miles. On the third day, they drove the remaining 200 miles.

4. Create a graph to represent Natasha's family trip. Be sure to label your axes with quantities and units and label specific points that highlight the trip.



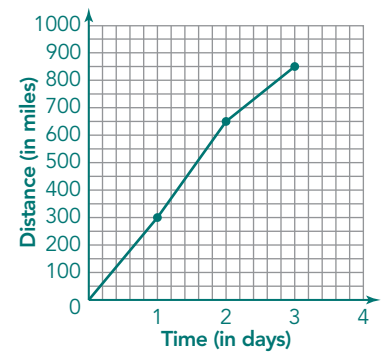
LESSON 3: There Are Many Paths ... • M4-95

Answers

3. Sample answer.

The graph describes the amount of money in a parking meter before and after it is emptied. The quantities that change are time in hours and the amount of money in quarters. The label for the x-axis is time, in hours. The label for the y-axis is money, in quarters.

4.



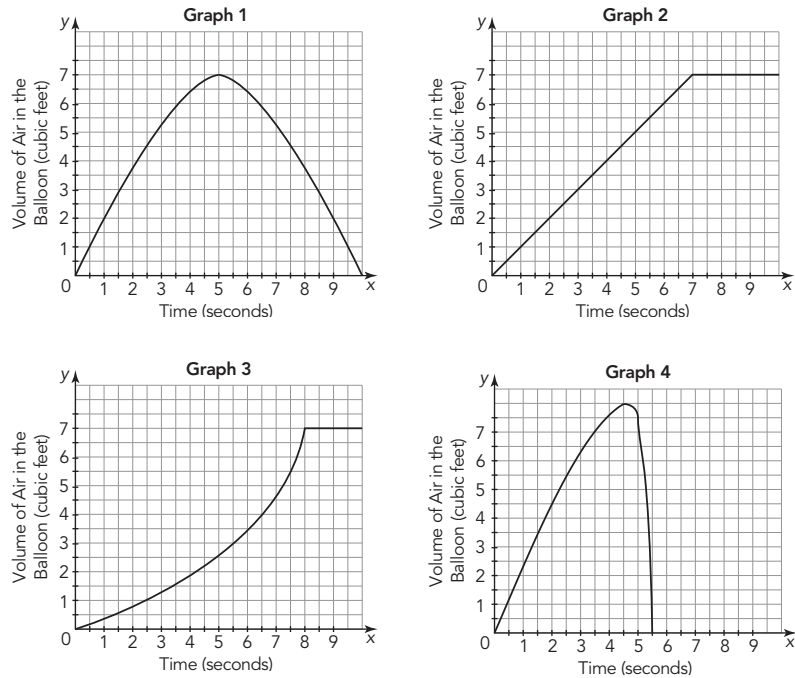
- What do you think it means when a graph goes up or down from left to right?
- What about when the graph goes flat?

Have students ask their own questions about the graphs. Encourage responses from fellow students, listening for correct interpretations.

Answers

- 5a. The x-axis represents the amount of time to blow up each balloon.
- 5b. The y-axis represents the volume of air in each balloon.
- 5c. Independent quantity is the amount of time spent blowing up the balloon. Dependent quantity is the amount of air in each balloon.

Nadja is coordinating the neighborhood Spring Fling. She asks Matthew to blow up balloons for the event. The graphs shown represent his efforts.



5. Analyze each graph shown, and then answer each question.

- What quantity is represented on the x-axis in each graph?
- What quantity is represented on the y-axis in each graph?
- Which quantity is independent quantity and which is dependent quantity?

6. Match each description with the appropriate graph.

a. Matthew blows air into a balloon at a steady rate, then ties it off when it is full.

b. Matthew blows air into a balloon, and then the balloon pops!

c. Matthew blows air into a balloon, and then lets the air out.

d. Matthew blows air into a balloon slowly. As the balloon stretches out, he is able to blow more air into the balloon. He then ties off the balloon when it is full.

Answers

6a. Graph 2

6b. Graph 4

6c. Graph 1

6d. Graph 3

Answers

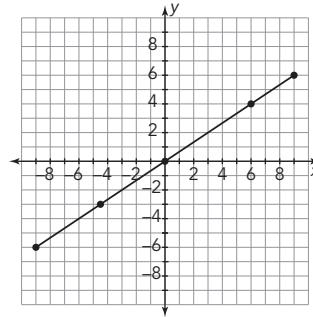
- The independent quantity, which is graphed on the x-axis, is the time since 3:00 PM (in hours). The dependent quantity, which is graphed on the y-axis, is the water level above or below the desired fill level (in inches).
- See table below.

ACTIVITY 3.4

Pool Level



The graph shows the water level of a pool. The x-axis represents time, in hours, and the y-axis represents the water level, in inches. The origin represents 3:00 P.M. and the desired water level.



- Label the graph with the independent and dependent quantities and their units.
- Create a table of values for the points plotted and describe the meaning of each.

x	y	Meaning

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x	y	Meaning
-9	-6	At 6:00 AM, the water level was 6 inches below the desired fill level.
-4.5	-3	At 10:30 AM, the water level was 3 inches below the desired fill level.
0	0	At 3:00 PM, the water level was perfect!
6	4	At 9:00 PM, the water level was 4 inches higher than desired.
9	6	At midnight, the water level was 6 inches higher than desired.

3. At what rate did the water go into the pool?
Explain your reasoning.

4. Describe a situation that would match the graph.

5. Write an equation for this situation.

6. Why does the graph stop rather than continue infinitely?

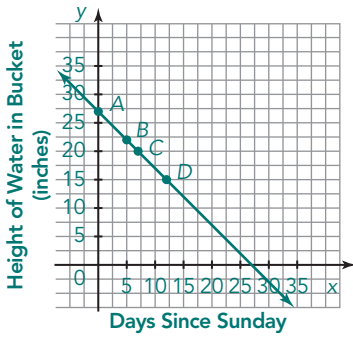
7. Using any of your mathematical tools, determine the time when the pool was 3 inches above the desired fill level. Is your answer exact or approximate? Explain.

Answers

- The water was added to the pool at a rate of 2 inches every 3 hours, or $\frac{2}{3}$ inch per hour.
- A person could have started filling a pool in the morning, and then have forgotten to turn off the hose, allowing the water level to rise above the preferred level.
- Let x represent the time in hours. Let y represent the water level in inches. $y = \frac{2}{3}x$
- Sample answers. At some point, possibly at 6 inches above desired level, the pool will be completely full and it will not be possible to fill it any more.
- Sample answers. Using the graph, the approximate time the water level is 3 inches above the desired level is 7:00 PM. Using the table, the water level is above 3 inches sometime between 3:00 PM and 9:00 PM. Using the equation, the exact answer is 4.5 hours after 3:00 PM, or exactly 7:30 PM.

Answers

1.



NOTES

ACTIVITY 3.5

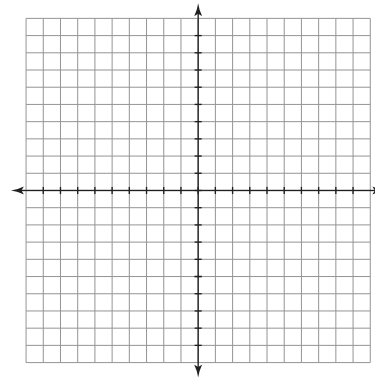
Water in the Bucket



As part of a science project, Damon collected water in a bucket in his backyard and is studying the evaporation. Unfortunately, Damon is a bit forgetful and forgets to take measurements of the water every day. The first day he remembered was Sunday, which was 4 days AFTER the data collection was to begin. He collects the following data.

Days Since Sunday	Height of Water (inches)
0	27
5	22
7	20
12	15

1. Graph the data. Connect the data values with a line. Be sure to label your axes.



2. Assuming that the water evaporated at the same rate every day, use your graph to determine the water level the day he was supposed to start data collection.

3. Assuming that the water evaporated at the same rate every day, use your graph to determine when the water level was

- | | |
|---------------|---------------|
| a. 30 inches. | b. 12 inches. |
| c. 5 inches. | d. 0 inches. |

4. Explain why you should or should not extend your graph into Quadrant IV.

Answers

2. 31 inches

3a. $x = -3$, which was Thursday before he started.

3b. $x = 15$, which was the Monday, fifteen days after he started.

3c. $x = 22$, which was the third Monday after he started.

3d. $x = 27$, which was a Saturday, almost 4 weeks after he started measuring.

4. The water level will never be negative, so we should not extend our graph into the Quadrant IV. We should stop at $(27, 0)$.

Answers

1.

Hours Worked	Pay (\$)
0	0
5	37.50
10	75
15	112.50
20	150

2. Because the graph is a straight line, passing through the origin, it represents equivalent ratios. The ratio of pay to hours worked is 75 : 10 or 7.5 dollars : 1 hour, which is also a rate.

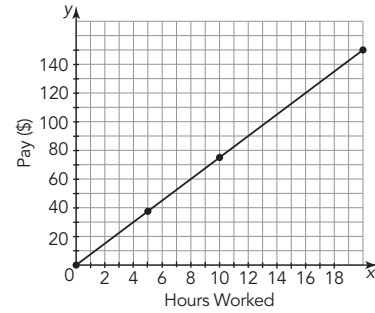
NOTES

ACTIVITY 3.6

Hard Working in the Hardware Store



Your friend Aidan got a job working at the local hardware store. He created the graph shown to track how much money he makes for a given number of hours.



1. Create a table of values for Aidan's graph.

Hours Worked	Pay (\$)
0	
5	
10	
15	
20	

2. How can you tell, by looking at the graph, whether the graph displays equivalent ratios? If it does, what is the ratio, or rate, displayed in the graph?

3. Define variables for the hours worked and Aidan's pay.
4. Write an equation to describe Aidan's graph.
5. Use the tool of your choice—equation, graph, or a table—to answer each equation.
- Approximately how much money did Aidan make if he worked 15 hours this week?
 - Determine the exact amount of money Aidan made if he worked 12 hours this week.
 - Approximately how many hours did Aidan work if he made \$50 this week?
 - Determine the exact number of hours Aidan worked if he made \$152.50 this week.
 - How did you decide which tool to use to answer each question?

Answers

3. Let h be the number of hours worked and let p be Aidan's pay.
4. $p = 7.5h$
- Sample answers.
- 5a. Using the graph, approximately \$110. Using the table, exactly \$112.50.
- 5b. \$90.00
- 5c. Around 7 hours
- 5d. 20 hours and 20 minutes
- 5e. If the question asked for "approximately," an estimate from the graph was sufficient. If the question asked for "exactly," I used the equation and solved for the unknown.

Answers

1. Yes. Lilianna is correct because the yardstick starts at $2\frac{1}{2}$ inches.
2. The length is $8\frac{1}{2}$ inches. The width is 7 inches.
3. Let b be equal to the measurement with the broken yardstick and a be equal to the actual measurement.
4. $b = a + 2\frac{1}{2}$ or $a = b - 2\frac{1}{2}$

ACTIVITY 3.7

Broken Yardstick



Jason and Lilianna need to measure some pictures so they can buy picture frames. They looked for something to use to measure the pictures, but could find only a broken yardstick. The yardstick was missing the first $2\frac{1}{2}$ inches.

They both thought about how to use this yardstick.

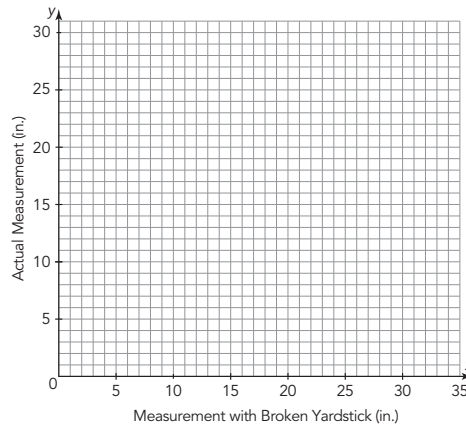
Lilianna said that all they had to do was measure the pictures and then subtract $2\frac{1}{2}$ inches from each measurement.

1. Is Lilianna correct? Explain your reasoning.
2. They measured the first picture's length and width to be 11 inches and $9\frac{1}{2}$ inches. What are the actual length and width?
3. Define variables for a measurement with the broken yardstick and the actual measurement.
4. Write an equation that models the relationship between the variables.

5. Complete the table of values for the measurement on the yardstick and the actual measurement.

Measurement with Broken Yardstick (in.)	Actual Measurement (in.)
11	
$9\frac{1}{2}$	
$25\frac{3}{4}$	
21	
$18\frac{5}{8}$	
	12
	$29\frac{1}{8}$
	$6\frac{7}{8}$

6. Use the table to complete the graph of the actual measurements versus the measurement taken with the broken yardstick.



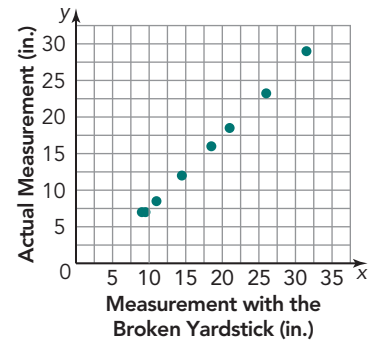
7. Would it make sense to connect the points on this graph? Explain why or why not.

Answers

5.

Measurement with Broken Yardstick (in.)	Actual Measurement (in.)
11	$8\frac{1}{2}$
$9\frac{1}{2}$	7
$25\frac{3}{4}$	$23\frac{1}{4}$
21	$18\frac{1}{2}$
$18\frac{5}{8}$	$16\frac{1}{8}$
$14\frac{1}{2}$	12
$31\frac{5}{8}$	$29\frac{1}{8}$
$9\frac{3}{8}$	$6\frac{7}{8}$

6.

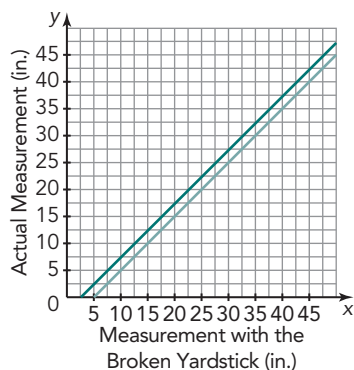


7. Yes. The numbers on the x-axis represent measurements, and you can have decimal or fractional measurements.

Answers

8a. $b = a + 5$ or $a = b - 5$

8b.



8c. The rate is the same on the two graphs. The new graph looks like the original one but shifted down $2\frac{1}{2}$ inches.

8d. The x-coordinate $(2\frac{1}{2}, 0)$ and $(5, 0)$ of these points tell us how many inches are broken off the broken yardstick. Those are the smallest measurements that can be made with each yardstick.

8. Suppose the yardstick was broken at 5 inches instead of $2\frac{1}{2}$ inches.

a. Write the new equation for the relationship between the actual measurement and the measurement from the broken yardstick.

b. Sketch a graph of the actual measurements versus the measurement taken with the new broken yardstick on the graph with the original yardstick.

c. What do you notice about the two graphs?

d. What is the meaning of the x-intercept—the point with a y-coordinate of zero—on each graph?

ACTIVITY
3.8

The Diver



A freediver is a person who dives into the ocean without the use of any breathing device like scuba equipment. William Trubridge holds the record for freediving. In 2016, he broke his own record and dove almost 407 feet into the ocean! Suppose you plan to train as a freediver and want to beat Trubridge's record.

1. What are some questions you would ask of Trubridge about his dive?

2. Assume that Trubridge ascended and descended at the same rate of 2.97 feet per second to help you determine how much time you need to be able to hold your breath to beat Trubridge's record.

Consider which representations would be useful—tables, graphs, or equations.



Answers

1. Sample answers.

- How long did you have to hold your breath to dive that far?
- How fast did you descend and ascend?
- Does it take the same amount of time to dive down as it does to come up?
- How did you train for this dive?

2. Let t represent the time he dives in seconds (the independent variable), and let d represent his depth in feet (the dependent variable).

Time (Seconds)	Depth (Feet)
10	29.7
30	89.1
60	178.2
90	267.3
120	356.4
150	445.5

The last row is too far, so it took less than 150 seconds to dive to a depth of 407 feet. So it would take less than 300 seconds, or 5 minutes, to complete his dive.

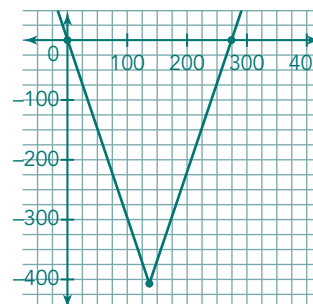
$$d = -2.97t \text{ for the descent}$$

$$407 = 2.97t$$

$$t = 137.04$$

After 137.04 seconds, or approximately 2 minutes 17 seconds, Trubridge would have reached a depth of 407 feet below the surface.

Assuming it takes the same amount of time to come back up, Trubridge had to hold his breath over 4 minutes 34 seconds! This graph would look like a V.



Answers

1. Sample answer.

At the state fair, I can purchase 3 ride tickets for \$4.

- $(0, 0)$ means if I do not buy any ride tickets, it will not cost any money.
- $(3, -4)$ means if I buy 3 ride tickets, it will cost \$4.
- $(6, -8)$ means if I buy 6 ride tickets, it will cost \$8.
- $(9, -12)$ means if I buy 9 tickets, it will cost \$12.
- $(12, -16)$ means if I buy 12 tickets, it will cost \$16.
- $(15, -20)$ means if I buy 15 tickets, it will cost \$20.
- Let t represent the number of ride tickets (the independent variable), and let m represent the amount of money I leave the fair with, in dollars (the dependent variable).
- $m = -4t$

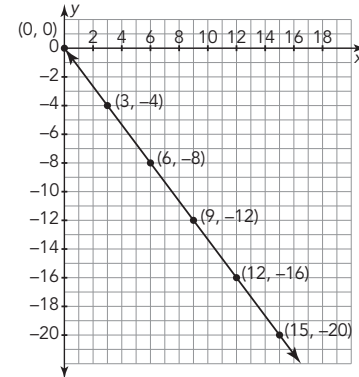
TALK the TALK

Your Turn!

You and your group should prepare a presentation for this problem.

1. **Create a situation that can be modeled by the graph.**

Write at least 3 sentences for what you want to say during your presentation.



- Be sure to determine the ratio, or rate, for how the variables change in relation to each other.
- Describe the meaning of each point on the graph.
- Define variables for the independent and dependent quantities based on your situation.
- Write an equation to represent the problem situation.

Assignment

Write

Give an example of when you might want to use an equation to answer a question and another example of when you might want to use a graph.

Remember

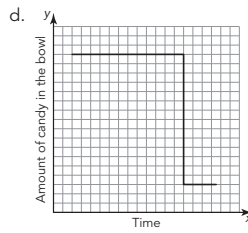
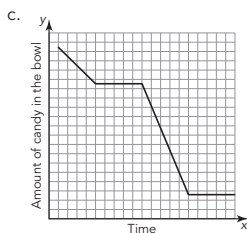
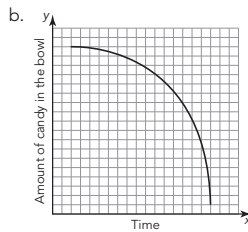
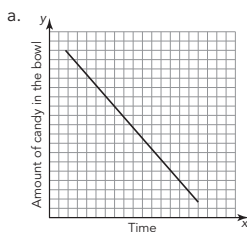
Graphs, tables, equations, and scenarios provide different information and allow for various levels of accuracy when solving problems.

Practice

- The gravitational pull of the Moon is not as great as that of Earth. In fact, if a person checks his weight on the Moon, it will be only $\frac{1}{6}$ of his weight on Earth.
 - If a person weighs 186 pounds on Earth, how much will he weigh on the Moon? How many pounds different from his actual weight is that?
 - Complete the table of values for a person's weight on Earth, weight on the Moon, and difference of the two weights. Use negative numbers when the weight is less than the person's earth weight.

Weight on Earth (lb)	186	168		198		
Weight on Moon (lb)			29		21	24
Weight Differential	-155					

- Graph the weight differential versus the weight on Earth. Be sure to label your axes.
- To keep her students relaxed and focused during tests, Ms. Chappell puts small bowls of candy on each of their desks. Write a short story to describe each graph.



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- This student did not eat any candy. At some point toward the end of the test, he accidentally knocked over the candy bowl and most of the candy fell out. He didn't eat any of the candy remaining in the bowl.

Assignment Answers

Write

Answers will vary.

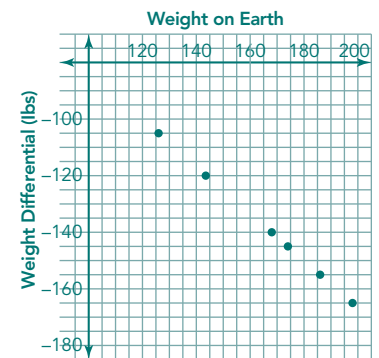
Practice

1a. 31 pounds; 155 pounds less

1b.

186	168	174	198	126	144
31	28	29	33	21	24
-155	-140	-145	-165	-105	-120

1c.



- This student ate the candies at a steady rate throughout the test.
- This student started eating the candy slowly and then increased the rate at which he ate the candy throughout the test.
- This student started eating the candy at a steady rate. He then stopped eating the candy for a short period. He then began eating the candy at a faster rate than before. After a little while, he stopped eating the candy.

Assignment Answers

3a. Independent quantity is the number of days since January 1, in days. The dependent quantity is the temperature, in degree Celsius, in Fairbanks, Alaska.

3b.

x	y	Meaning
-8	-4	8 days before January 1, the average temperature was -4 degrees Celsius.
-2	-1	2 days before January 1, the average temperature was -1 degree Celsius.
0	0	On January 1, the average temperature was 0 degrees Celsius.
4	2	4 days after January 1, the average temperature was 2 degrees Celsius.
6	3	6 days after January 1, the average temperature was 3 degrees Celsius.

3c. The temperature increased at a rate of 1 degree every 2 days, or $\frac{1}{2}$ degree per day.

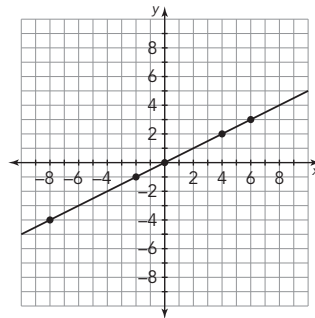
3d. Let x represent time in days from January 1. Let y represent the temperature in Fairbanks, Alaska. $y = \frac{1}{2}x$

4a. 3 pounds; $12\frac{3}{4}$ pounds

4b. Let w represent the number of weeks Bruno has been on the diet (the independent variable), and let p represent the number of pounds that Bruno loses (the dependent variable).

4c. $p = -1\frac{1}{2}w$

3. The following graph shows the average temperature, in degrees Celsius, in Fairbanks, Alaska. The x -axis represents time in days from January 1, and the y -axis represents degrees Celsius.



- Label the axes with the independent and dependent quantities and their units.
- Create a table of values for the points on the graph and describe the meaning of each.

x	y	Meaning

- At what rate did the temperature increase?
- Define variables for the quantities that are changing, and write an equation for this situation.

4. Sarina's dog, Bruno, has to go on a diet! Sarina puts Bruno on a diet plan of daily exercise and a special type of dog food. She estimates Bruno will lose $1\frac{1}{2}$ pounds per week on this plan.

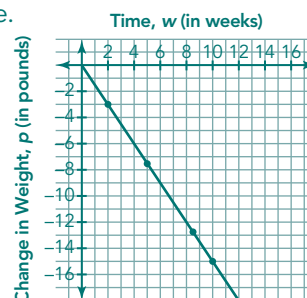
- How many pounds does Sarina estimate Bruno will lose in 2 weeks? In $8\frac{1}{2}$ weeks?
- Define variables for the independent and dependent quantities for this situation.
- Write an equation for this situation. (Because Bruno is losing weight, the number of pounds he loses will be defined as a negative value.)
- Create a table of values for the situation.
- Complete a graph of the situation.
- Explain what points in Quadrant I would mean for Bruno.

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4d.

Independent Quantity	Dependent Quantity
Number of Weeks	Change in Weight
weeks	pounds
w	p
2	-3
5	-7.5
8.5	-12.75
10	-15

4e.



4f. Bruno would be gaining weight, not losing it.

Assignment Answers

Stretch

Sample answer.

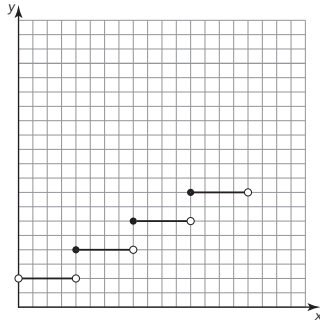
Your telephone calling card charges \$0.40 for the first minute of calls and \$0.40 for each additional minute of calls.

Review

1. The area is 35.5 square units.
- 2a. Answers will vary. The low temperature today was -3°F and the high was 21°F . The temperature changed 24°F .
- 2b. Answers will vary. I began an 8 hour work shift and I have completed the first 3 hours of the shift. I have 5 hours remaining of the work shift.
- 3a. 9; 22.52
- 3b. $\frac{13}{12}$; $\frac{33}{20}$

Stretch

Tell a story to describe the graph.



Review

1. The vertices of a polygon are given. Plot the points on a coordinate plane and connect the points in the order they are listed. Then determine the area of the polygon.
 $(-4, -1), (-3, -2), (10, -2), (3, 0), (0, 4), (-2, 3)$
2. Create a scenario to fit each numeric expression.
 - a. $|-3 + 21|$
 - b. $|8 - 3|$
3. Evaluate each expression for the given values.
 - a. $5.2r + 1.2$, when $r = 1.5$ and 4.1
 - b. $\frac{1}{2}t + \frac{3}{4}$, when $t = \frac{2}{3}$ and $\frac{9}{5}$

