What's in a Name?

Rational Number System

3

Lesson Overview

Students formally classify numbers as rational numbers and understand that all numbers they have studied so far are subsets of the rational numbers. Students sort and classify numbers. They investigate the density of rational numbers by locating rational numbers between other rational numbers.

Grade 6 The Number System

Apply and extend previous understandings of numbers to the system of rational numbers.

7. Understand ordering and absolute value of rational numbers.
   c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation.
   d. Distinguish comparisons of absolute value from statements about order.

Essential Ideas

- Rational numbers are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and b does not equal 0.
- The set of rational numbers includes the sets of integers, whole numbers, and natural numbers.
- Given two rational numbers, there exists an infinite number of rational numbers between those numbers.

Lesson Structure and Pacing: 1 Day

Engage

Getting Started: Sort It Out!

Students cut out and sort numbers into different student-generated groups. The class creates a list of ways they sorted the numbers.

Develop

Activity 3.1: Analyzing Number Sorts

Students compare number sorts completed by other students. These sorts lead to the formal introduction of the set of rational numbers. Students regroup their cards into the set of Natural Numbers and then Whole Numbers. The terms *Integers* and *Rational Numbers* are formally defined. Students identify the cards that belong to each number set.

Activity 3.2: Classifying Numbers

Students play Number Set Bullseye to classify numbers into the number sets defined in Activity 3.1. Then they complete a table that includes some descriptions and examples of the different sets of numbers.

Activity 3.3: Density

Students order and compare numbers. The Density Property is stated and students use this property to identify a rational number between a pair of rational numbers on a number line. They will plot various positive and negative decimals, whole numbers, fractions and mixed numbers on the same number line. They then plot and label a rational number between a pair of given rational numbers.

Demonstrate

Talk the Talk: Do They Always Belong?

Students summarize what they learned in the lesson by evaluating statements about the relationship between different sets of numbers. Students determine if each statement is true or false and explain their reasoning.

Facilitation Notes

In this activity, students cut out and sort numbers into different student-generated groups. The class creates a list of ways they sorted the numbers.

Have a student read the instructions aloud. Then work with a partner or in groups to complete Question 1.

Differentiation strategy

For students who struggle,
- Reduce the number of cards to be sorted.
- Have them simplify some of the numbers on the cards by writing equivalent numbers (without the percent signs, double signs or absolute value symbols) along with the original numbers.

Questions to ask
- How did you group the numbers?
- What are the criteria for belonging to each group in your sort?
- Is there another way you could sort the numbers?
- Do any of your numbers fit into multiple groups?

As a class, complete Question 2. Generate a list of groups created by the students. This is a brainstorm. It is not necessary for the list to be exhaustive.

Summary

Numbers can be classified in a variety of ways.

Activity 3.1
Analyzing Number Sorts

DEVELOP

Facilitation Notes

In this activity, students compare number sorts completed by other students, which leads to the formal introduction of the set of rational numbers. Cards are regrouped into the set of natural numbers and then whole numbers. The terms *integers* and *rational numbers* are defined and used to regroup numbers.

Have students work with a partner or in groups to complete Questions 1 through 3.

Questions to ask
- What do you notice about the numbers Suzanne grouped together?
- What might Suzanne think about the numbers she grouped together?
- Compare the numbers not in Suzanne's group with the numbers in the group. How are the groups different?
- What is common about the numbers Zane chose to omit from his group?
- What title do you think Zane gave his group?
- Does Amelia's sort meet the requirements of the sort instructions? Explain.
- How would you write the numbers in Suzanne's group as fractions?

Differentiation strategies

To assist all students, suggest they take notes as they work through Questions 4 through 6.
- In the margin by Question 4, write the definitions of natural numbers and whole numbers.
- Highlight the definition of integers.
- Highlight the definition of rational numbers.

Have a student read the statement before Question 4 aloud. Ask students to work with a partner or in groups to complete Question 4. Share responses as a class.

Differentiation strategies
- As students learn about the different number sets, they will create a larger and larger pile of number cards until all of the cards are in a single pile. This forming of groups is intended to reinforce that number sets are nested, or are subsets of each other.
- Discuss the definitions of *natural numbers* and *whole numbers* by first asking students if they know what kinds of numbers are natural numbers and what kinds of numbers are whole numbers. This should be prior knowledge. The set of natural numbers is the same as the set of counting numbers. The set of whole numbers is the set of natural numbers along with 0.

Questions to ask
- Does $|-452|$ represent a natural number? A whole number?
- Does $-(-9)$ represent a natural number? A whole number?
- How are natural numbers and whole numbers different?
- Which set of numbers is a subset of the other?
- How are your piles of natural numbers and whole numbers the same? Different?

Have a student read the definition of integers aloud. Students should highlight the definition of integers. Have students complete Question 5 with a partner or in groups. Share responses as a class.

Questions to ask
- Does $|-452|$ represent an integer?
- Does -5.78 represent an integer?
- Does -452 represent an integer?
- Are negative whole numbers also integers?
- Are numbers written as decimals also integers?
- Are numbers written as fractions also integers?
- Are numbers written as percents also integers? What about 100%?
- How are integers and whole numbers different?
- How are your piles of integers and whole numbers the same? Different?

Have a student read the definition of rational numbers aloud. Have students complete Question 6 with a partner or in groups. Share responses as a class.

Questions to ask
- Does $\left|\frac{215}{16}\right|$ represent a rational number?
- Does -5.78 represent a rational number?
- Does 0.25% represent a rational number?
- Are integers also rational numbers?
- Are numbers written as decimals also rational numbers?
- Are numbers written as fractions also rational numbers?
- Are numbers written as percents also rational numbers?
- How are integers and rational numbers different?
- How are your piles of integers and rational numbers the same? Different?

Misconception
Students sometimes confuse the common meaning of a word with its mathematical meaning. In this case, students are familiar with the definition of rational being reasonable. Connect the mathematical definition of rational with ratio.

Differentiation strategy
To support all students in understanding the definition of rational numbers, have them write all the numbers on the cards as rational numbers in the form $\frac{a}{b}$.

Discuss how the pile(s) changed as the students created the pile of natural numbers, whole numbers, integers, and finally rational numbers.

Questions to ask
- What happened to the previous pile of numbers when you created a new pile?
- What do your piles mean about how the number sets are related to each other?

Summary
Numbers can be classified into number sets using definitions.

Activity 3.2
Classifying Numbers

Facilitation Notes

In this activity, students use Number Set Bullseye to classify numbers into the number sets defined in Activity 3.1. They also complete a table that includes some descriptions and examples of the different sets of numbers.

In preparation for the lesson, create a Number Set Bullseye on the board or on poster paper. Use note cards to create number cards. Copy the numbers provided in Question 1. Have tape available to attach the numbers to the bullseye.

Have a student read the information before Question 1 aloud. Make sure that all students understand the meaning of *subset*.

Ask students to work with a partner or in groups to complete Question 1. To play pin the number on the bullseye, distribute the number cards that match the numbers in Question 1. Have students attach them one-by-one to the bullseye. Allow the class to provide feedback. Have the class check their answers and make corrections on the table in Question 1. Allow students to create their own number cards and challenge their classmates.

Questions to ask
- How do you decide if a number is in the set of rational numbers? Integers? Whole numbers? Natural numbers?
- What is the difference between the set of natural numbers and the set of whole numbers?
- What is the difference between the set of whole numbers and the set of integers?
- What is the difference between the set of integers and the set of rational numbers?

- What is an example of a number that is both a natural number and a whole number?
- What is an example of a number that is both a whole number and an integer?
- What is an example of a number that is both a natural number and an integer?
- Are all natural numbers whole numbers or are all whole numbers natural numbers?
- Are all integers whole numbers or are all whole numbers integers?
- Does the set of natural numbers contain the set of whole numbers?
- Does the set of whole numbers contain the set of natural numbers?
- Does the set of whole numbers contain the set of integers?
- Does the set of integers contain the set of whole numbers?
- Does the set of integers contain the set of rational numbers?
- Does the set of rational numbers contain the set of integers?

Have students complete Question 2 individually. Share responses as a class.

Encourage students to provide a wide range of examples for the rational numbers.

Summary

Numbers can belong to more than one number set, and number sets can be nested inside each other.

Activity 3.3
Density

Facilitation Notes

In this activity, students order and compare numbers. The Density Property is introduced and used to identify a rational number between a pair of rational numbers on a number line. Positive and negative decimals, whole numbers, fractions and mixed numbers are located on the same number line.

Have a student read the paragraph about the Density Property aloud.

Questions to ask
- Are the terms dense and density related? How?
- How is the word dense used in other situations?
- What does the term density mean in science?
- What does the term density mean in health? (bone density)
- What does a sign reading 'Densely Populated Area Ahead' mean?

- What are rational numbers? How are they different from the other sets of numbers?
- What is special about rational numbers that makes them dense?
- How did you informally learn about the density of rational numbers when you learned about fractions in elementary school?
- How did you informally learn about the density of rational numbers when you learned about decimals in elementary school?
- Why doesn't the Density Property hold true for natural numbers?
- Why doesn't the Density Property hold true for whole numbers?
- Why doesn't the Density Property hold true for integers?

Have students work with a partner or in groups to complete Question 1. Share responses as a class.

Differentiation strategies
- Assign groups of students to create Human Number Lines that correspond to each part (a), (b), (c), (d). Students create the number line with masking tape but need to fit all group members (and more!) between the designated numbers. Students may decide to "zoom in" their number lines more than what is provided in the text.
- If students work with a partner, have all pairs of students write their numbers on the board for all 4 parts. Challenge the students to generate numbers they don't think their classmates will generate.

Questions to ask
- What is a rational number between $4\frac{1}{3}$ and $4\frac{1}{2}$?
- What is a rational number between 5.51 and 5.52?
- What is a rational number between 0.451 and 0.452?
- What is a rational number between -0.458 and -0.459?
- Is it possible for the absolute value of a number to be equal to zero?
- Which number is located furthest to the left on the number line?
- Which number is located furthest to the right on the number line?
- What strategy are you using to determine your answers?

Have students work with a partner to complete Questions 2 through 4. Share responses as a class.

Differentiation strategy
Create teams of students. Each team needs a writing tool and a mini white board, note card, or blank piece of paper. When the first problem is called, give each team the same amount of time to write a number that meets the criterion. At the same time, all teams hold up their numbers. Discuss the results. The team with the number closest to 1, 0, −5, respectively, wins a point. Repeat for each question and add additional questions.

Questions to ask
- What is a rational number between _____ and _____ ?
- How can you write a number even closer to _____ ?

- Why is your number closer to _____ than your partner's number?
- How long did it take you to develop a strategy for the game?
- Did you modify your strategy as you played the game? How?

Summary

Between any two rational numbers is an infinite number of rational numbers.

Talk the Talk: Do They Always Belong?

Facilitation Notes

In this activity, students evaluate statements about the relationship between different sets of numbers and explain their reasoning.

Have students complete Questions 1 through 6 individually. Share responses as a class.

Questions to ask
- How do you decide if a number is in the set of rational numbers? Integers? Whole numbers? Natural numbers?
- Are all rational numbers whole numbers or all whole numbers rational numbers?
- Are all integers rational numbers or are all rational numbers integers?
- Are all integers whole numbers or are all whole numbers integers?

As students work, look for students who are confused about the nesting of the sets of numbers. Direct them to the Number Set Bullseye and to the definitions of the sets of numbers.

Summary

Numbers can belong to more than one number set, and number sets can be nested inside each other.

What's in a Name?

Rational Number System

3

WARM UP

Represent each decimal or percent as a fraction in lowest terms.

a. 0.3

b. 2.8

c. $\frac{3}{4}\%$

d. 212%

LEARNING GOALS

- Classify numbers according to their number systems.
- Apply and extend an understanding of whole numbers and integers to the system of rational numbers.
- Understand ordering of rational numbers.

KEY TERMS

- integers
- ellipsis
- rational numbers
- Density Property

You use many different types of numbers in math class and in the world, including whole numbers, fractions, and decimals, both positive and negative. How can you organize and classify different types of numbers?

Answers

Students may sort the cards into a variety of categories: positive versus negative, integers and non-integers, opposites, numbers with the same absolute value, numbers with the same value, odd and even numbers.

Sort It Out!

Cut out the cards found at the end of the lesson. Then, analyze and sort the numbers into different groups. You may group them in any way you feel is appropriate, but you must sort the numbers into more than one group.

1. For each of your groups,

   • create a title that fits the numbers in that group.
   • list the numbers included.
   • write a rationale for why you group those particular numbers.

2. Compare your sort with your classmates' sorts. Create a list of the different ways your class grouped the numbers.

ACTIVITY 3.1 — Analyzing Number Sorts

1. **Suzanne grouped these numbers together. Why do you think she put these numbers in the same group?**

$$0, -452, 9, 24, |-3|, -3, -(-9), |-452|$$

2. **Zane had a group similar to Suzanne's but he did not include −452 and −3. Why do think Zane omitted these numbers from his group?**

3. **Amelia said that she created two groups: Group 1 contains all the numbers that can be written as fractions and Group 2 contains all the numbers that cannot be written as fractions. Analyze Amelia's sorting idea.**

   a. **Which numbers do you think Amelia placed in Group 2?**

   b. **Justine is not sure about Amelia's sort. She thinks that all of the numbers can be written as fractions. Is Justine correct? Explain why or why not.**

Answers

1. The numbers are all integers.

2. Zane included only numbers that represented positive values, or whole numbers.

Sample answers.

3a. She may think only numbers written as fractions, percents, and decimals can be written as fractions. $0, -452, 24, 9, -3, |-3|, -(-9), |-452|$

3b. Justine is correct. All numbers in the sort can be written as fractions. All integers can be written as fractions with a denominator of 1.

You have used different sets of numbers, including the set of natural, or counting, numbers and the set of whole numbers.

4. Identify the numbers from the sort that are in each set.

   a.   natural numbers

   b.   whole numbers

> Notice the three periods before and after specific numbers in the set. These three periods are called an **ellipsis,** and they are used to represent infinity in a number set.

Throughout this topic, you have been learning about the set of *integers*. **Integers** are the set of whole numbers with their opposites. The integers can be represented by the set {…, −3, −2, −1, 0, 1, 2, 3, …}.

5. Identify the numbers from the sort that are included in the set of integers.

You have also worked with rational numbers throughout this year. **Rational numbers** are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

6. Identify the numbers from the sort that are included in the set of rational numbers.

Classifying Numbers

There are many ways you can classify numbers. As you saw in the previous activity, many of the classifications are subsets of other classifications. The diagram shows the different sets of numbers you have encountered in your mathematical experiences.

Natural numbers are a subset of whole numbers.

Whole numbers are a subset of integers.

Integers are a subset of rational numbers.

Pin the number on the bullseye! Your teacher will direct students to pin (or tape) a number card to its correct location in the diagram of the rational number set.
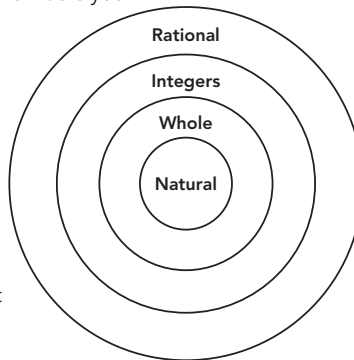
1. **For each value, check all of the number sets to which it belongs.**

Number	Natural Number	Whole Number	Integer	Rational Number
3				
3.222				
0				
−4.5				
$-\frac{3}{5}$				
54				
−5				
$\frac{23}{3}$				
0.667				
−1,364,698				

Answers

During the activity, students should pin each number inside the ring of the bullseye that is the most inclusive number system in which the number belongs.

1.

Natural Number	Whole Number	Integer	Rational Number
X	X	X	X
			X
	X	X	X
			X
			X
X	X	X	X
		X	X
			X
			X
		X	X

ELL Tip

Have English Language Learners take turns speaking and listening to another student speak about *whole*, *natural*, *integer*, and *rational numbers*. The student speaking gives an example and a non-example for each system, and shares his or her opinion about whether the name of the system makes sense.
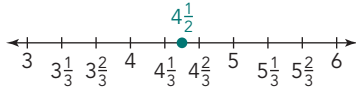
Answers

2. Sample answers.

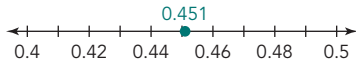| Whole Numbers	Integers	Rational Numbers
0, 1, 2, 3, ...		$\frac{1}{2}$, 0.35, 5
	Whole numbers with their opposites	Numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$

Answers

Sample answers.

1a.

1b.

1c.

1d.

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2. Complete the table with the missing examples and descriptions.

|	Natural Numbers	Whole Numbers	Integers	Rational Numbers
Examples	1, 2, 3, ...		..., −3, −2, −1, 0, 1, 2, 3, ...	
Description	Counting numbers	Natural numbers and 0		

ACTIVITY 3.3

Density

The **Density Property** states that between any two rational numbers there is another rational number. The property is not true for natural numbers, whole numbers, or integers. For example, there is no integer between 25 and 26. There is no whole number or natural number between 12 and 13.

1. Plot the given rational numbers. Then plot and label a rational number between each pair of rational numbers.
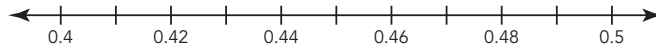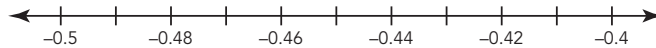
a. $4\frac{1}{3}$ and $4\frac{2}{3}$
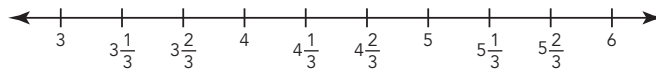
b. 5.5 and 5.6
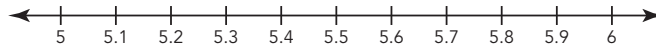
c. 0.45 and 0.46

d. −0.45 and −0.46

Complete each rational number line with a partner.

2. **Create a number line from 0 to 1. Your goal is to plot and label a rational number closer to 1 than your partner.**

   <--->

   Partner 1: Plot a rational number, *A*, between 0 and 1 that is close to 1.

   Partner 2: Plot a rational number, *B*, between *A* and 1.

   Repeat at least 2 more times.

3. **Create a number line from −1 to 0. Your goal is to plot and label a rational number closer to 0 than your partner.**

   <--->

   Partner 1: Plot a rational number, *A*, between −1 and 0 that is close to 0.

   Partner 2: Plot a rational number, *B*, between *A* and 0.

   Repeat at least 2 more times.

4. **Create a number line from −6 to −5. Your goal is to plot and label a rational number closer to −5 than your partner.**

   <--->

   Partner 1: Plot a rational number, *A*, between −6 and −5 that is close to −5.

   Partner 2: Plot a rational number, *B*, between *A* and −5.

   Repeat at least 2 more times.

Answers

Answers will vary based on students' choices of numbers.

2. The goal is for students to determine rational numbers between 0 and 1 but closer to 1. For example, $\frac{9999}{10,000}$ or 0.9999.

3. The goal is for students to determine negative rational numbers close to 0. For example, $-\frac{1}{10,000}$.

4. The goal is for students to determine rational numbers between −6 and −5 but closer to −5. For example, −5.000001.

Answers

1. True. All whole numbers can be written with a denominator of 1, they are all rational numbers by definition.

2. False. Rational numbers include numbers, such as $\frac{2}{3}$, which are not in the set of whole numbers.

3. False. Rational numbers include numbers, such as $\frac{2}{3}$, which are not in the set of integers.

4. True. All integers can be written with a denominator of 1, they are all rational numbers by definition.

5. True. The integers, by definition, include all whole numbers and their opposites, so all whole numbers are integers.

6. False. Integers include the opposites of whole numbers and those opposites are not whole numbers.

TALK the TALK

Do They Always Belong?

Determine if each statement is true or false. Justify your answer using definitions and/or examples.

1. True   False   All whole numbers are rational numbers.

2. True   False   All rational numbers are whole numbers.

3. True   False   All rational numbers are integers.

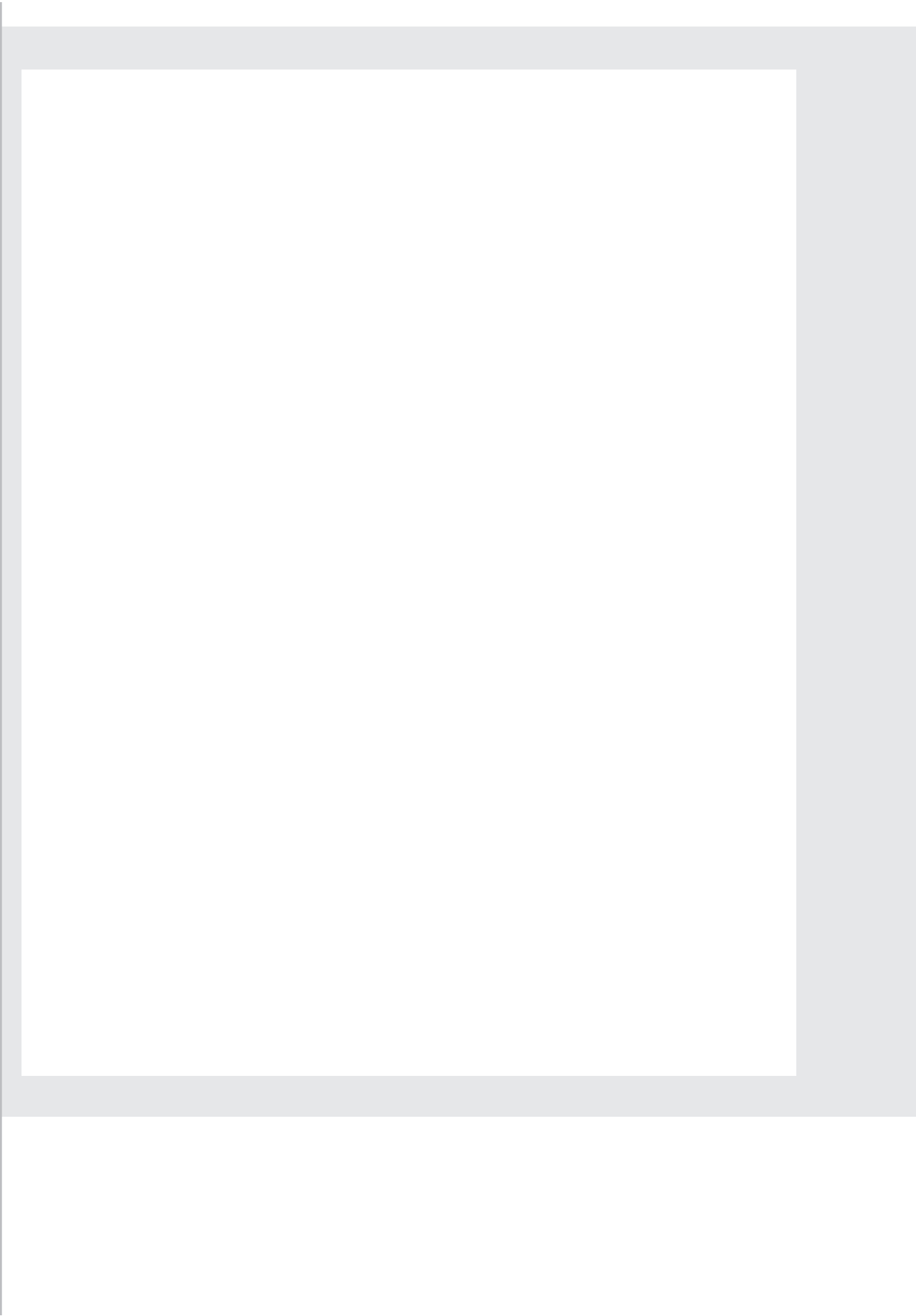4. True   False   All integers are rational numbers.

5. True   False   All whole numbers are integers.

6. True   False   All integers are whole numbers.

0	-5.78	$2\frac{15}{16}$	$\frac{3}{4}\%$	-452
$\frac{1}{2}$	24	9	$\frac{6}{7}$	$-\frac{6}{7}$
-0.5	0.5	$-\frac{1}{2}$	2.5%	5.78
-3	$|-3|$	$-\frac{2}{3}$	$\frac{1}{1000}$	0.001
-6.41	$|6.41|$	$-(-9)$	$|-452|$	-0.3
225%	$6\frac{1}{4}$	25%	0.25%	$\left|\frac{215}{16}\right|$

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Assignment

Write

Define each term in your own words.

1. The set of rational numbers
2. The Density Property

Remember

Rational numbers include all numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not zero.
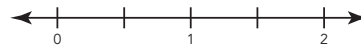
Practice

1. Write all the sets of numbers to which each value belongs.
   a. The tundra covers about $\frac{1}{5}$ of Earth's surface.
   b. The average annual temperature is $-18°$ Fahrenheit.
   c. There are 48 varieties of land mammals found in the tundra region.
   d. The permafrost is a layer of frozen soil that is located below Earth's surface at -1476 feet.
   e. During the summer months, the low temperature averages about 37.4° F.

2. Nadine collects data about some animals. Determine a rational number between each pair of rational numbers. Plot all three numbers on a number line.
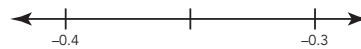   a. A mole's runway is between -3 and -12 inches in the ground.
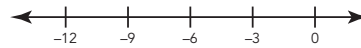
   b. The musky rat kangaroo weighs between $\frac{3}{4}$ and $\frac{3}{2}$ pound.

   c. The percent of change of the Alaskan polar bear population in the past year was between -0.33 and -0.32.

Assignment Answers

Write

Answers will vary.

Practice

1a. Rational numbers

1b. Rational numbers, integers

1c. Rational numbers, integers, whole numbers, natural numbers

1d. Rational numbers, integers

1e. Rational numbers

2. Answers will vary. Check students' number lines. Examples provided.

2a. -3.75

2b. $1\frac{1}{8}$

2c. -0.325

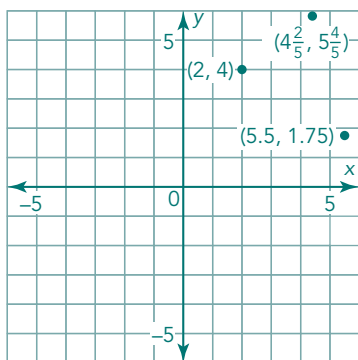## Assignment Answers

Stretch

Answers will vary.

Review

1a. $|-10| + |40| = 50$; the temperature changed by 50°.

1b. $|-30| + |200| = 230$; you have hiked 230 feet. you have hiked 230 feet.

2.

m	w
27.6	3
46	5
74.52	8.1
92	10
128.8	14

3a, 3b, 3c.

Stretch

Are there more integers or more natural numbers? Even though there are infinitely many of both, it seems like there should be more integers than natural numbers. But, actually, there are just as many integers as there are natural numbers!

If you can show how to assign an integer to every natural number, you will demonstrate that the two sets of numbers are equal. How do you think this can be done?

Review

1. Write an absolute value expression to calculate the answer to each question.
   a. The temperature at 9:00 A.M. was 40°. The temperature at 2:00 P.M. was −10°. What was the change in temperature?
   b. You began your hike at 30 feet below sea level. You are now at 200 feet. How far have you hiked?

2. Complete the table for the equation $w = \frac{m}{9.2}$.

m	w
27.6	
	5
74.52	
92	
	14

3. Plot each ordered pair on a coordinate plane.
   a. (2, 4)
   b. (5.5, 1.75)
   c. $(4\frac{2}{5}, 5\frac{4}{5})$