3

What's in a Name?

Rational Number System

WARM UP

Represent each decimal or percent as a fraction in lowest terms.

a. 0.3

b. 2.8

c. $\frac{3}{4}\%$

d. 212%

LEARNING GOALS

- Classify numbers according to their number systems.
- Apply and extend an understanding of whole numbers and integers to the system of rational numbers.
- Understand ordering of rational numbers.

KEY TERMS

- integers
- ellipsis
- rational numbers
- Density Property

You use many different types of numbers in math class and in the world, including whole numbers, fractions, and decimals, both positive and negative. How can you organize and classify different types of numbers?

Sort It Out!

Cut out the cards found at the end of the lesson. Then, analyze and sort the numbers into different groups. You may group them in any way you feel is appropriate, but you must sort the numbers into more than one group.

1. **For each of your groups,**

   - **create a title that fits the numbers in that group.**
   - **list the numbers included.**
   - **write a rationale for why you group those particular numbers.**

2. **Compare your sort with your classmates' sorts. Create a list of the different ways your class grouped the numbers.**

ACTIVITY
3.1

Analyzing Number Sorts

NOTES

1. Suzanne grouped these numbers together. Why do you think she put these numbers in the same group?

   $0, -452, 9, 24, |-3|, -3, -(-9), |-452|$

2. Zane had a group similar to Suzanne's but he did not include -452 and -3. Why do think Zane omitted these numbers from his group?

3. Amelia said that she created two groups: Group 1 contains all the numbers that can be written as fractions and Group 2 contains all the numbers that cannot be written as fractions. Analyze Amelia's sorting idea.

   a. Which numbers do you think Amelia placed in Group 2?

   b. Justine is not sure about Amelia's sort. She thinks that all of the numbers can be written as fractions. Is Justine correct? Explain why or why not.

You have used different sets of numbers, including the set of natural, or counting, numbers and the set of whole numbers.

4. Identify the numbers from the sort that are in each set.

a. natural numbers

b. whole numbers

"Notice the three periods before and after specific numbers in the set. These three periods are called an **ellipsis**, and they are used to represent infinity in a number set."

Throughout this topic, you have been learning about the set of *integers*. **Integers** are the set of whole numbers with their opposites. The integers can be represented by the set {..., −3, −2, −1, 0, 1, 2, 3, …}.

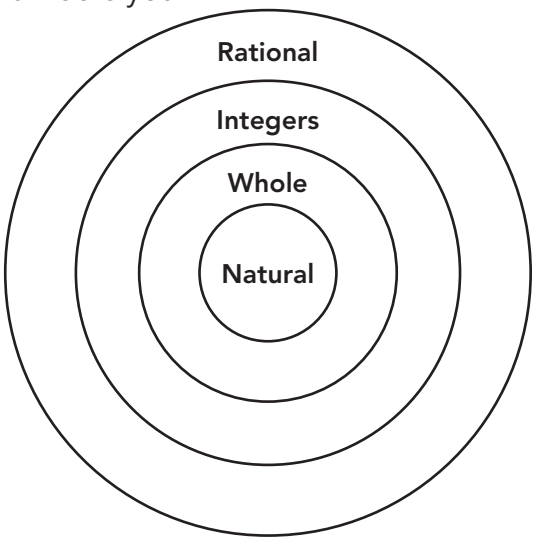
5. Identify the numbers from the sort that are included in the set of integers.

You have also worked with rational numbers throughout this year. **Rational numbers** are the set of numbers that can be written as $\frac{a}{b}$, where a and b are integers and $b \neq 0$.

6. Identify the numbers from the sort that are included in the set of rational numbers.

ACTIVITY
3.2 Classifying Numbers

There are many ways you can classify numbers. As you saw in the previous activity, many of the classifications are subsets of other classifications. The diagram shows the different sets of numbers you have encountered in your mathematical experiences.



Natural numbers are a subset of whole numbers.

Whole numbers are a subset of integers.

Integers are a subset of rational numbers.

Pin the number on the bullseye! Your teacher will direct students to pin (or tape) a number card to its correct location in the diagram of the rational number set.

1. For each value, check all of the number sets to which it belongs.

Number	Natural Number	Whole Number	Integer	Rational Number
3				
3.222				
0				
−4.5				
$-\frac{3}{5}$				
54				
−5				
$\frac{23}{3}$				
0.667				
−1,364,698				

2. Complete the table with the missing examples and descriptions.

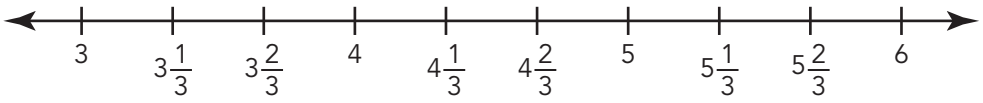
	Natural Numbers	Whole Numbers	Integers	Rational Numbers
Examples	1, 2, 3, ...		..., −3, −2, −1, 0, 1, 2, 3, ...	
Description	Counting numbers	Natural numbers and 0		

ACTIVITY
3.3 Density

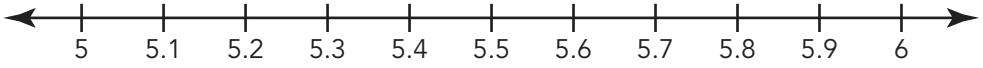
The **Density Property** states that between any two rational numbers there is another rational number. The property is not true for natural numbers, whole numbers, or integers. For example, there is no integer between 25 and 26. There is no whole number or natural number between 12 and 13.

1. Plot the given rational numbers. Then plot and label a rational number between each pair of rational numbers.

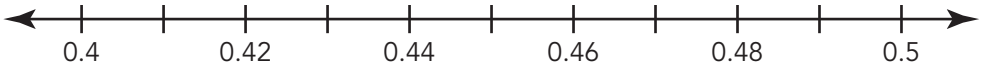
a. $4\frac{1}{3}$ and $4\frac{2}{3}$



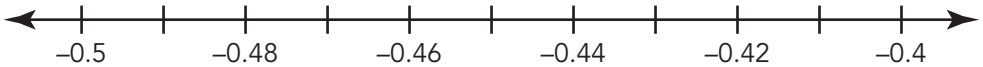
b. 5.5 and 5.6



c. 0.45 and 0.46



d. −0.45 and −0.46



Complete each rational number line with a partner.

2. Create a number line from 0 to 1. Your goal is to plot and label a rational number closer to 1 than your partner.

Partner 1: Plot a rational number, *A*, between 0 and 1 that is close to 1.

Partner 2: Plot a rational number, *B*, between *A* and 1.

Repeat at least 2 more times.

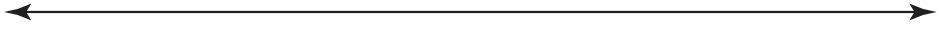
3. Create a number line from −1 to 0. Your goal is to plot and label a rational number closer to 0 than your partner.

Partner 1: Plot a rational number, *A*, between −1 and 0 that is close to 0.

Partner 2: Plot a rational number, *B*, between *A* and 0.

Repeat at least 2 more times.

4. Create a number line from −6 to −5. Your goal is to plot and label a rational number closer to −5 than your partner.



Partner 1: Plot a rational number, *A*, between −6 and −5 that is close to −5.

Partner 2: Plot a rational number, *B*, between *A* and −5.

Repeat at least 2 more times.

NOTES

TALK the TALK

Do They Always Belong?

Determine if each statement is true or false. Justify your answer using definitions and/or examples.

1. True False All whole numbers are rational numbers.

2. True False All rational numbers are whole numbers.

3. True False All rational numbers are integers.

4. True False All integers are rational numbers.

5. True False All whole numbers are integers.

6. True False All integers are whole numbers.

0	-5.78	$2\frac{15}{16}$	$\frac{3}{4}\%$	-452
$\frac{1}{2}$	24	9	$\frac{6}{7}$	$-\frac{6}{7}$
-0.5	0.5	$-\frac{1}{2}$	2.5%	5.78
-3	$\lvert -3 \rvert$	$-\frac{2}{3}$	$\frac{1}{1000}$	0.001
-6.41	$\lvert 6.41 \rvert$	$-(-9)$	$\lvert -452 \rvert$	-0.3
225%	$6\frac{1}{4}$	25%	0.25%	$\left\lvert \frac{215}{16} \right\rvert$

Assignment

Write

Define each term in your own words.

1. The set of rational numbers
2. The Density Property

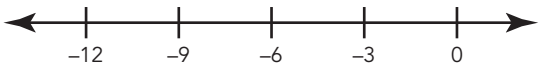
Remember

Rational numbers include all numbers that can be written in the form $\frac{a}{b}$, where a and b are integers and b is not zero.

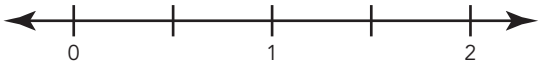
Practice

1. Write all the sets of numbers to which each value belongs.
   a. The tundra covers about $\frac{1}{5}$ of Earth's surface.
   b. The average annual temperature is $-18°$ Fahrenheit.
   c. There are 48 varieties of land mammals found in the tundra region.
   d. The permafrost is a layer of frozen soil that is located below Earth's surface at -1476 feet.
   e. During the summer months, the low temperature averages about 37.4° F.

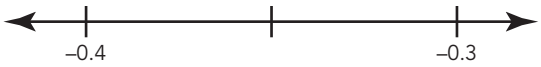
2. Nadine collects data about some animals. Determine a rational number between each pair of rational numbers. Plot all three numbers on a number line.
   a. A mole's runway is between -3 and -12 inches in the ground.



   b. The musky rat kangaroo weighs between $\frac{3}{4}$ and $\frac{3}{2}$ pound.



   c. The percent of change of the Alaskan polar bear population in the past year was between -0.33 and -0.32.



Stretch

Are there more integers or more natural numbers? Even though there are infinitely many of both, it seems like there should be more integers than natural numbers. But, actually, there are just as many integers as there are natural numbers!

If you can show how to assign an integer to every natural number, you will demonstrate that the two sets of numbers are equal. How do you think this can be done?

Review

1. Write an absolute value expression to calculate the answer to each question.
   a. The temperature at 9:00 A.M. was 40°. The temperature at 2:00 P.M. was −10°. What was the change in temperature?
   b. You began your hike at 30 feet below sea level. You are now at 200 feet. How far have you hiked?

2. Complete the table for the equation $w = \frac{m}{9.2}$.

m	*w*
27.6	
	5
74.52	
92	
	14

3. Plot each ordered pair on a coordinate plane.
   a. (2, 4)
   b. (5.5, 1.75)
   c. $(4\frac{2}{5}, 5\frac{4}{5})$